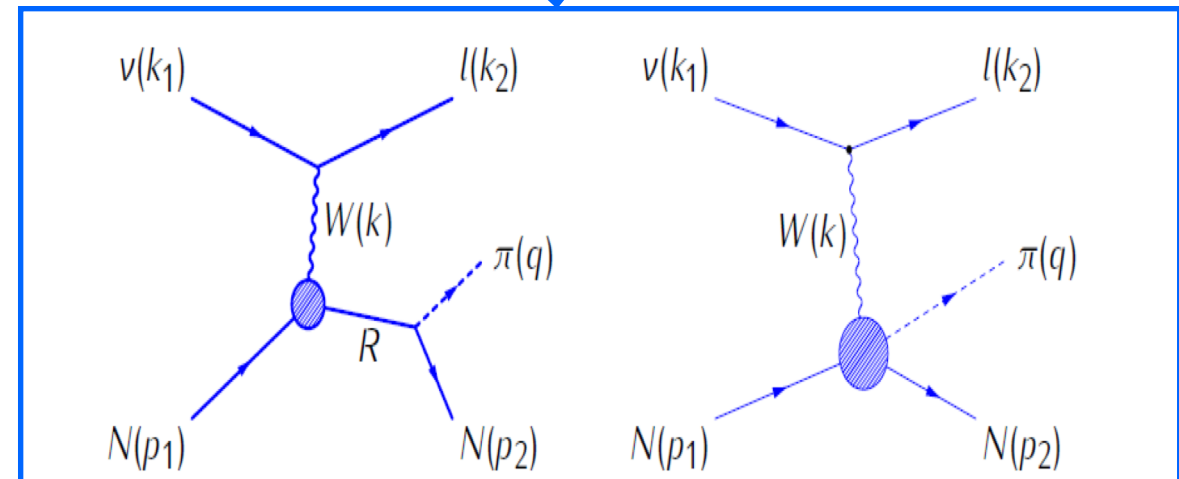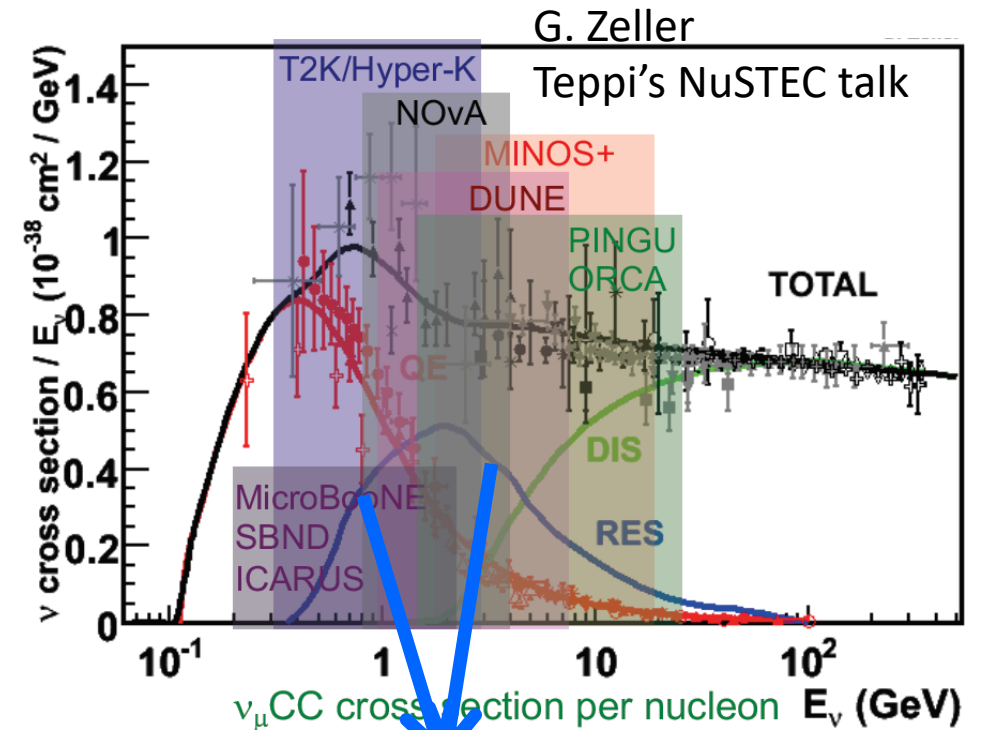# MK-model improvement

Minoo Kabirnezhad

GENIE meeting

Feb. 11, 2019

Single pion production

- Single pion can be produced via decay of resonance excitations or non-resonant interactions.

From
C. Wilkinson

Inclusive electron scattering data

- For E_ν <1 GeV only Δ resonance contributes but for higher energy (DUNE) all resonances contribute to single pion production.

Δ(1232) region
(1.08GeV<W<1.4 GeV)

- Δ resonance dominates

- Only single pion can be produced
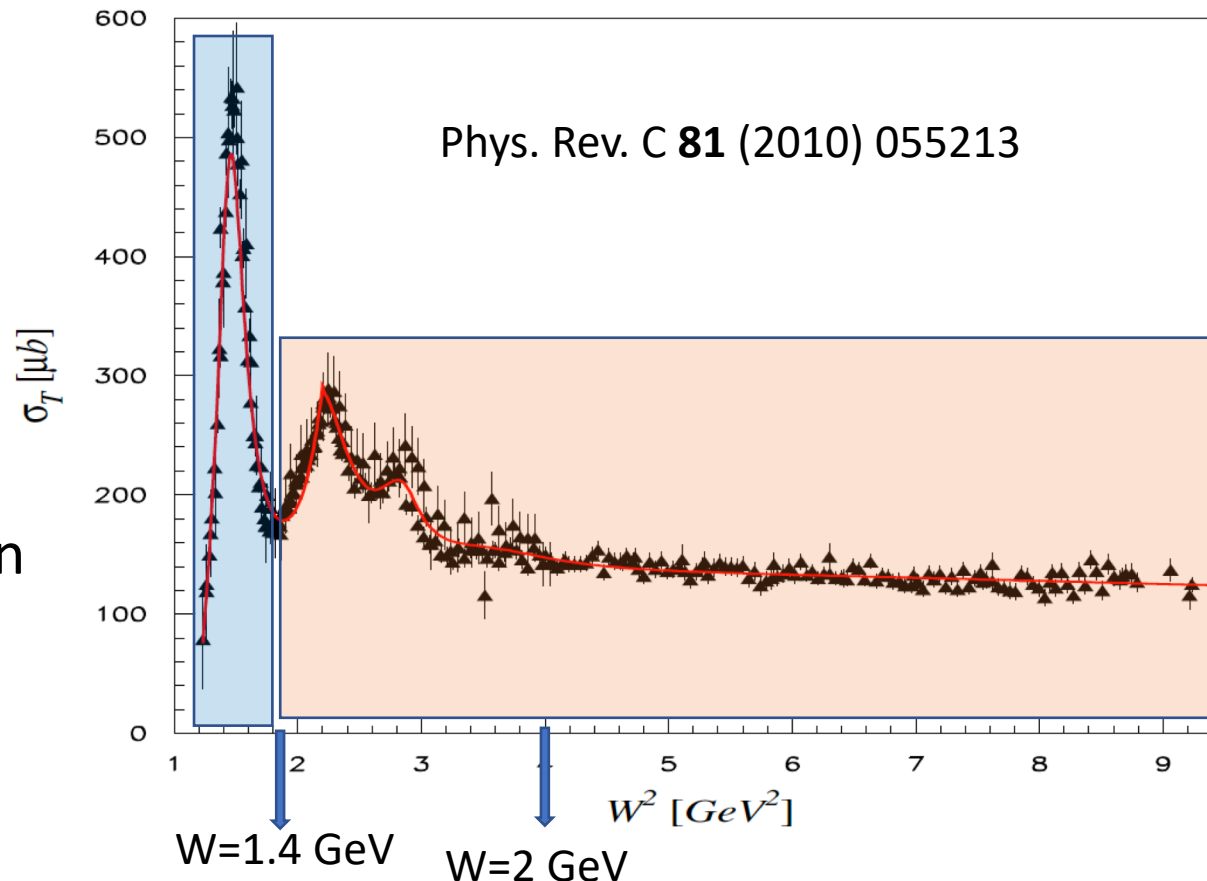
Beyond Δ region
W>1.4 GeV

- No single resonance dominate

- Several comparable resonances overlap

- Multi-pion and other mesons can be produced

Phys. Rev. C **81** (2010) 055213

$\sigma_T [\mu b]$

$W^2 [GeV^2]$

W=1.4 GeV

W=2 GeV

Rein-Sehgal model (1981)

D. Rein and L. M. Sehgal, Annals Phys. 133 (1981) 79.

Rein-Sehgal is default model in the **NEUT** and **GENIE**

👍 Easy to be implemented in generators.

👍 It covers all resonances up to 2 GeV.

👎 It does not cover non-resonant interaction

👎 Not a full kinematic model. $d\,\sigma/dW\,dQ^2$
The helicity amplitudes are **not** a function of pion angles

👎 Pion angles are described by density matrix.
NEUT and GENIE **only** implemented the Δ resonance.

The RS model is improved by including the pion angles and non-resonant interactions

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

Rein-Sehgal model (1981)

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Annals Phys. 133 (1981) 79.

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The helicity amplitudes are **not** a function of pion angles

👎 Pion angles are described by density matrix.
NEUT and GENIE **only** implemented the Δ resonance.

Output of the modified RS model
$$d\,\sigma/dW\,dQ^2\,d\Omega_\pi$$

| Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
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$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

MK-model (past)

- MK model is a model for single pion production
  i.e. resonant and nonresonant interactions including
  the interference effects.

- Uses Rein-Sehgal model to describe resonant interaction
  (17 resonances) up to W=2 GeV.

- Lepton mass is included.

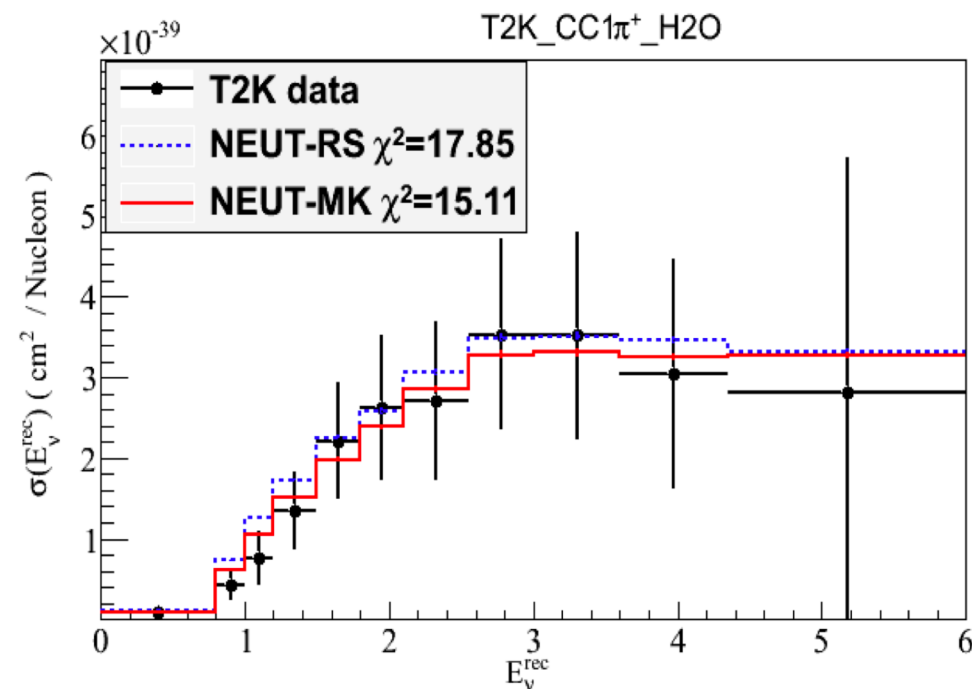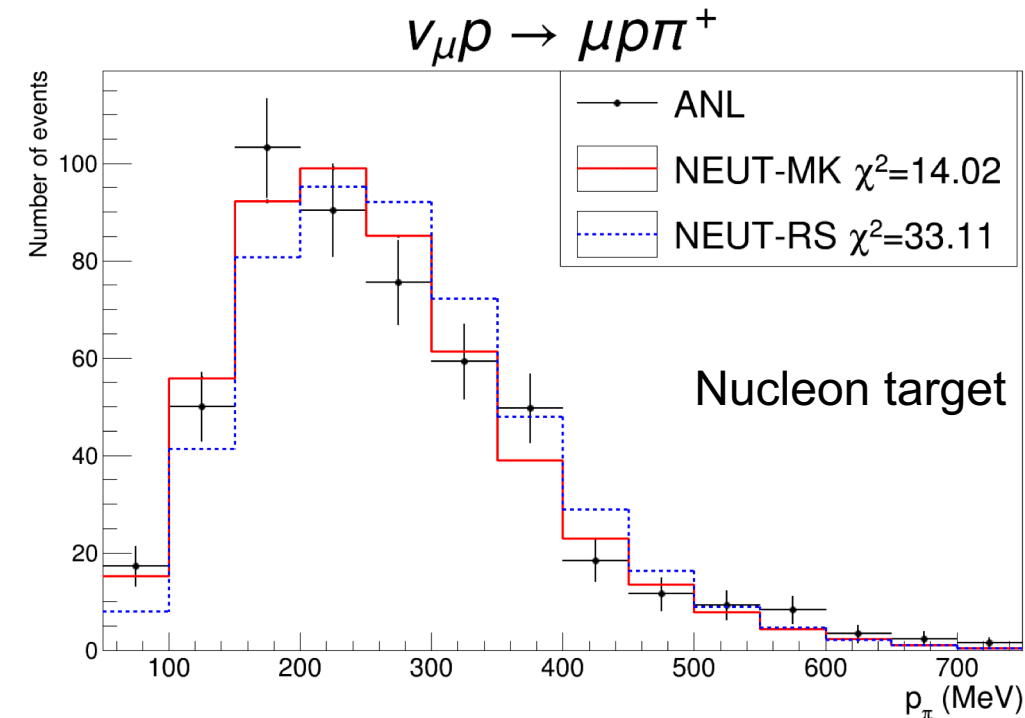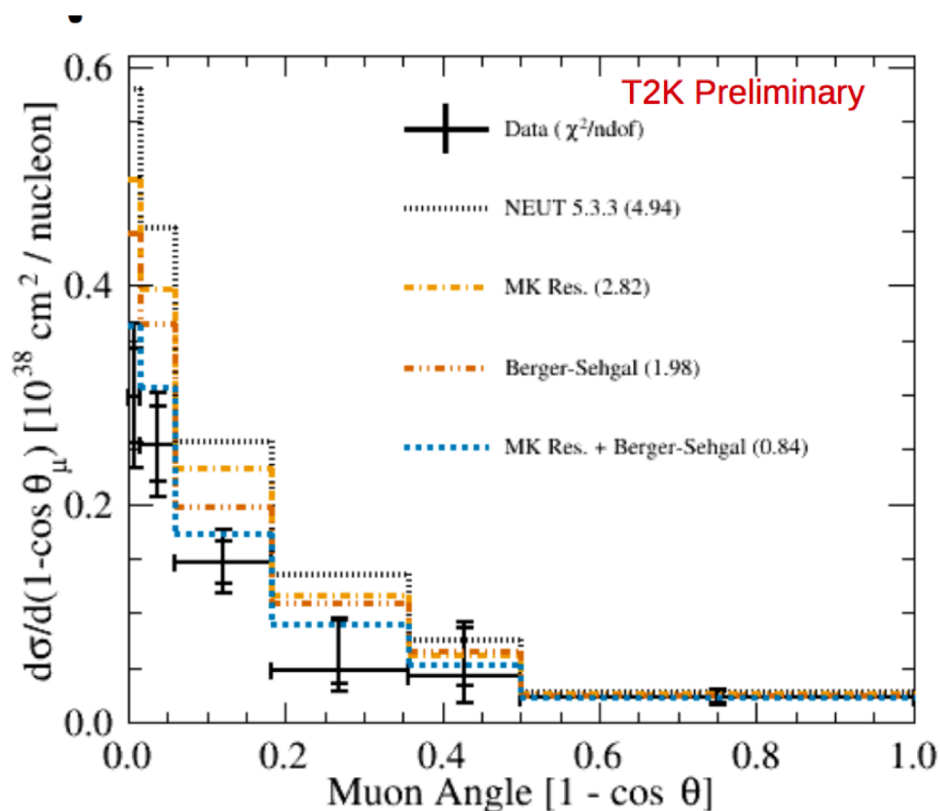- Non-resonant background is defined by a set of diagrams
  determined by HNV model.
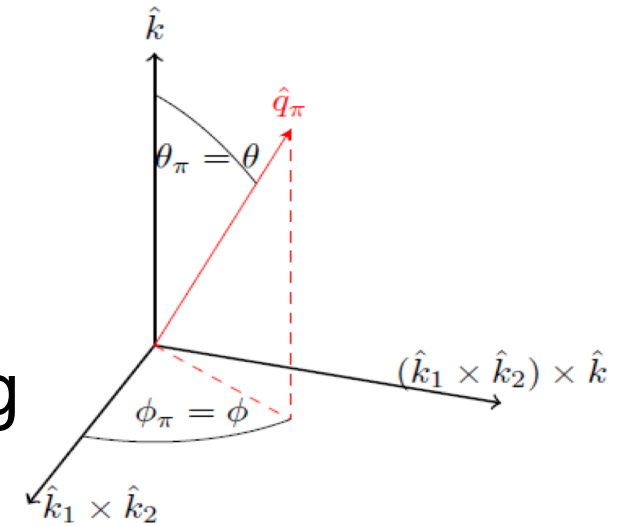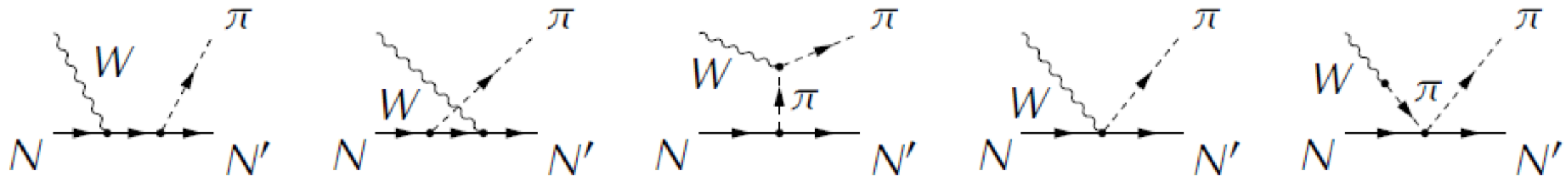
The MK-model in NEUT

NEUT comparisons with data
shows improvement with MK-model.

$$\nu_\mu p \to \mu p \pi^+$$

Legend:
- ANL
- NEUT-MK χ^2=14.02
- NEUT-RS χ^2=33.11

Nucleon target

p_π (MeV)

T2K Preliminary

- Data (χ^2/ndof)
- NEUT 5.3.3 (4.94)
- MK Res. (2.82)
- Berger-Sehgal (1.98)
- MK Res. + Berger-Sehgal (0.84)

Muon Angle [1 - cos θ]

$d\sigma/d(1-\cos\theta_\mu)$ [10^{38} cm^2 / nucleon]

T2K_CC1π^+_H2O

- T2K data
- NEUT-RS χ^2=17.85
- NEUT-MK χ^2=15.11

$\sigma(E_\nu^{rec})$ (cm^2 / Nucleon)

E_ν^{rec}

MK-model improvement

- **Verifying the model is difficult with limited neutrino data sets!**

- Existing neutrino data on "free" nucleon are old and with large error and it is very unlikely to be improved. ☹

A practical solution is to split the model

1. Vector part (electron scattering)

2. Axial part (pion scattering)

MK-model improvement
Vector part

1. Updating vector form-factor

- MK model used to have two form-factors (RS & GS)
- GS form-factor used to be the default form-factor.
- GSK is the updated form-factor for MK model. (see next slide)

2. fitting phases between resonance and non-resonance amplitudes with electron scattering data.

Graczyk-Sobczyk form-factor

- They equivalent the RS model with Lalakiluch et al model (Rarita-Schwinger formalism)

$$G_V^{RS}(Q^2, W) = \frac{1}{2\sqrt{3}} \left(1 + \frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}} \left[C_4^V \frac{W^2 - Q^2 - M^2}{2M^2} + C_5^V \frac{W^2 + Q^2 - M^2}{2M^2} + \frac{C_3^V}{M}(W+M)\right],$$

$$G_V^{RS}(Q^2, W) = -\frac{1}{2\sqrt{3}} \left(1 + \frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}} \left[C_4^V \frac{W^2 - Q^2 - M^2}{2M^2} + C_5^V \frac{W^2 + Q^2 - M^2}{2M^2} - C_3^V \frac{(M+W)M + Q^2}{MW}\right]$$

$$0 = C_4^V \frac{W}{M^2} + \frac{C_5^V}{M} \frac{(M+W)}{W} + \frac{C_3^V}{M}.$$

GS use the Lalakulich fit to e.m. data

A "partial" solution used by other models is:

$$C_5^V = 0, \quad C_3^V = -\frac{W}{M} C_4^V$$

$$C_4^V(Q^2) = -4\sqrt{3} \left(\frac{M}{M+W}\right)^2 \left(1 + \frac{Q^2}{(M+W)^2}\right)^{-3/2} G_V^{RS}(Q^2).$$

$$C_3^V = 2.13 \left(1 + \frac{Q^2}{4M_V^2}\right)^{-1} \left(1 + \frac{Q^2}{M_V^2}\right)^{-2},$$

$$C_4^V = -1.51 \left(1 + \frac{Q^2}{4M_V^2}\right)^{-1} \left(1 + \frac{Q^2}{M_V^2}\right)^{-2},$$

$$C_5^V = 0.48 \left(1 + \frac{Q^2}{4M_V^2}\right)^{-1} \left(1 + \frac{Q^2}{0.776M_V^2}\right)^{-2}$$

it does not agree well with the existing electromagnetic data.

Is there a typo in C_5?

Graczyk-Sobczyk form-factor

- They equivalent the RS model with Lalakiluch et al model (Rarita-Schwinger formalism)

$$G_V^{RS}(Q^2, W) = \frac{1}{2\sqrt{3}} \left(1 + \frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}} \left[C_4^V \frac{W^2 - Q^2 - M^2}{2M^2} + C_5^V \frac{W^2 + Q^2 - M^2}{2M^2} + \frac{C_3^V}{M}(W+M)\right],$$

$$G_V^{RS}(Q^2, W) = -\frac{1}{2\sqrt{3}} \left(1 + \frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}} \left[C_4^V \frac{W^2 - Q^2 - M^2}{2M^2} + C_5^V \frac{W^2 + Q^2 - M^2}{2M^2} - C_3^V \frac{(M+W)M + Q^2}{MW}\right]$$

$$0 = C_4^V \frac{W}{M^2} + \frac{C_5^V}{M} \frac{(M+W)}{W} + \frac{C_3^V}{M}.$$

A "partial" solution used by other models is:

$$C_5^V = 0, \quad C_3^V = -\frac{W}{M}C_4^V$$

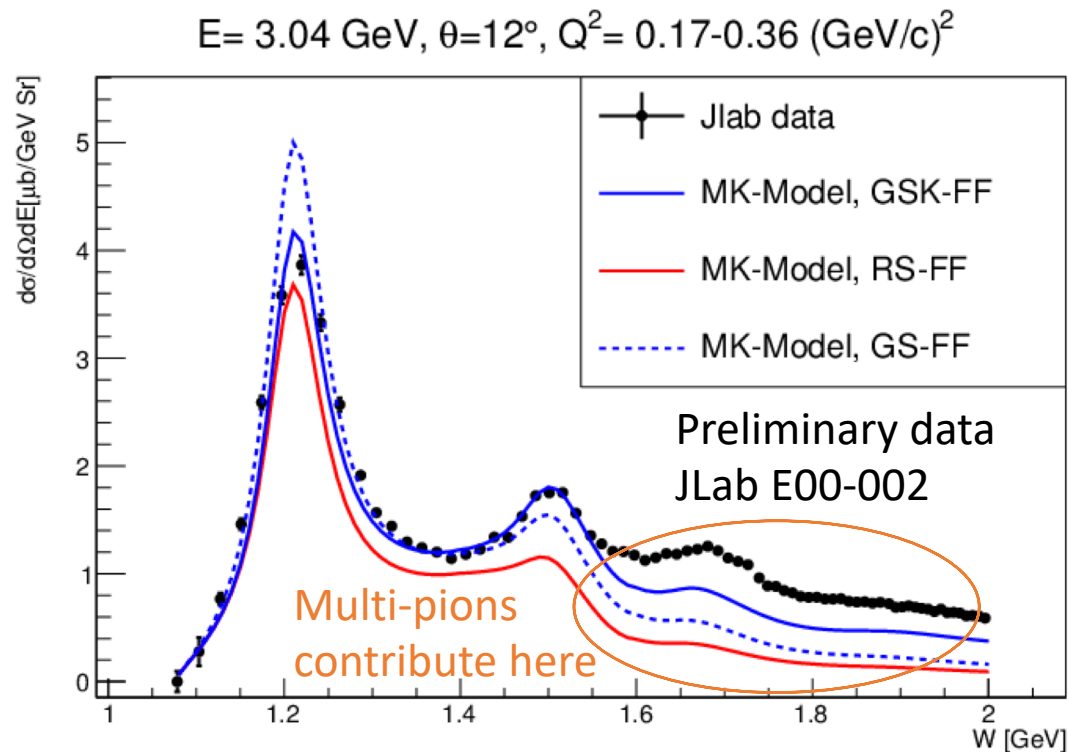$$C_4^V(Q^2) = -4\sqrt{3} \left(\frac{M}{M+W}\right)^2 \left(1 + \frac{Q^2}{(M+W)^2}\right)^{-3/2} G_V^{RS}(Q^2).$$

it does not agree well with the existing electromagnetic data.
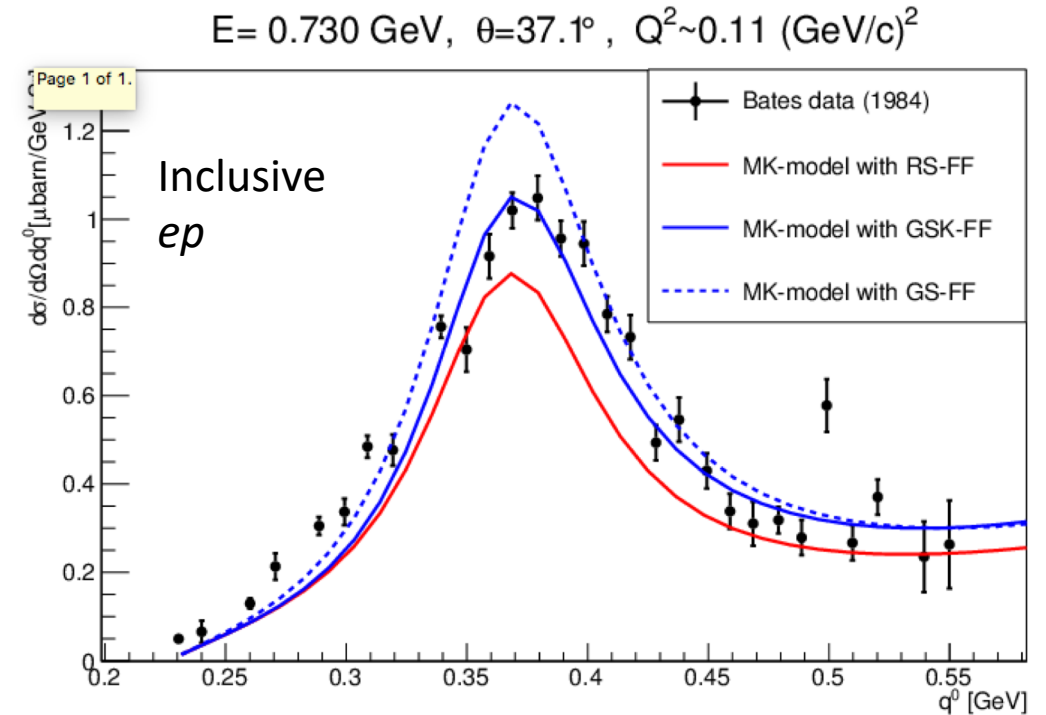
We should check the actual solution within MK-model

GSK form-factor

MK-model improvement (Vector part)

- MK-model with Graczyk-Sobczyk (GS) form factor does not agree with inclusive electron scattering data.

E= 3.04 GeV, $\theta=12°$, Q^2= 0.17-0.36 $(GeV/c)^2$

Jlab data

MK-Model, GSK-FF

MK-Model, RS-FF

MK-Model, GS-FF

Preliminary data
JLab E00-002

Multi-pions contribute here

More data comparison in backup

E= 0.730 GeV, $\theta=37.1°$, Q^2~0.11 $(GeV/c)^2$

Inclusive
ep

Bates data (1984)

MK-model with RS-FF

MK-model with GSK-FF

MK-model with GS-FF

- Vector form factor is updated version of GS form factor. It is called "GSK" form factor to distinguish.

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Cross-section definition in electron scattering

$$\frac{d\sigma_{em}}{d\Omega' dE' d\Omega_\pi^*} = \Gamma_{em} \left\{ \sigma_T + \varepsilon\sigma_L + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT} \cos\phi_\pi^* \right.$$

$$\left. + h\sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'} \sin\phi_\pi^* + \varepsilon\sigma_{TT} \cos 2\phi_\pi^* \right\}$$

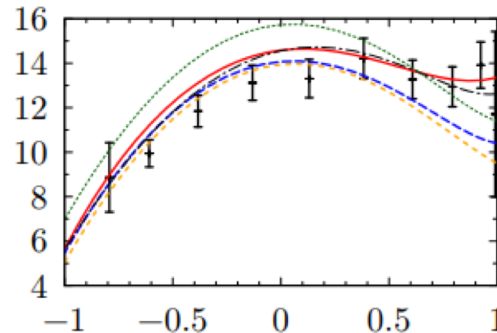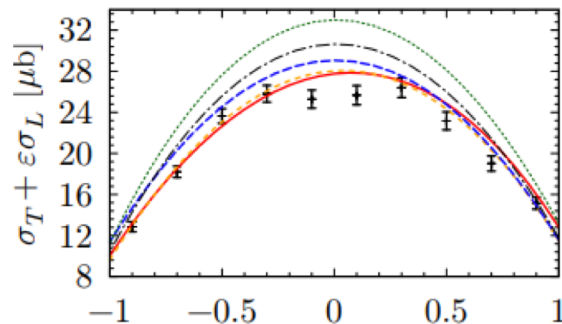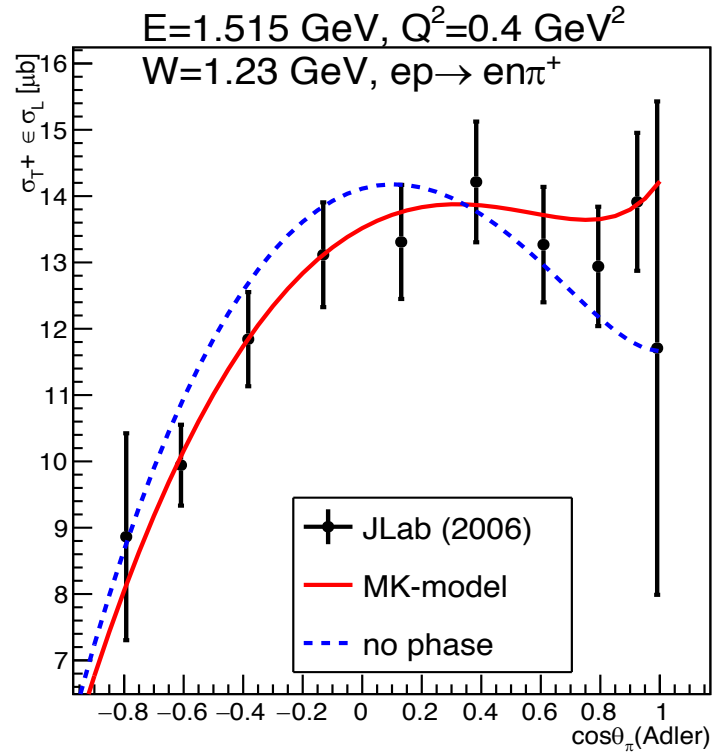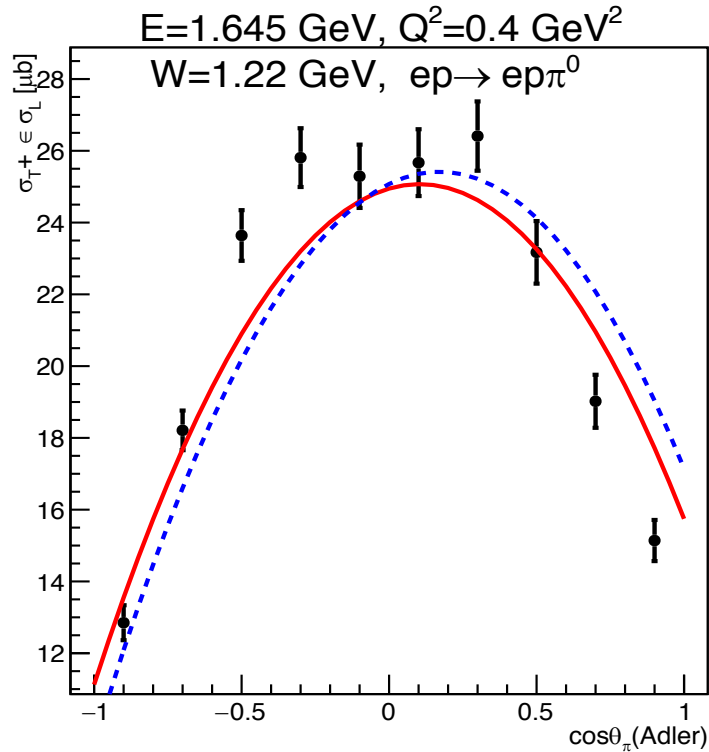$$\Gamma \equiv \frac{\alpha}{2\pi^2} \frac{E'}{E} \frac{(W^2 - m_p^2)}{2m_p Q^2} \frac{1}{1-\epsilon}$$

$$\epsilon \equiv \left(1 + 2\frac{|\mathbf{q}|^2}{Q^2} \tan^2 \frac{\theta_e}{2} \right)^{-1},$$

Γ is virtual photon flux factor

New Parameters:

1. A coefficient to form-factor of individual resonances.

2. A phase between resonance and bkg amplitudes.

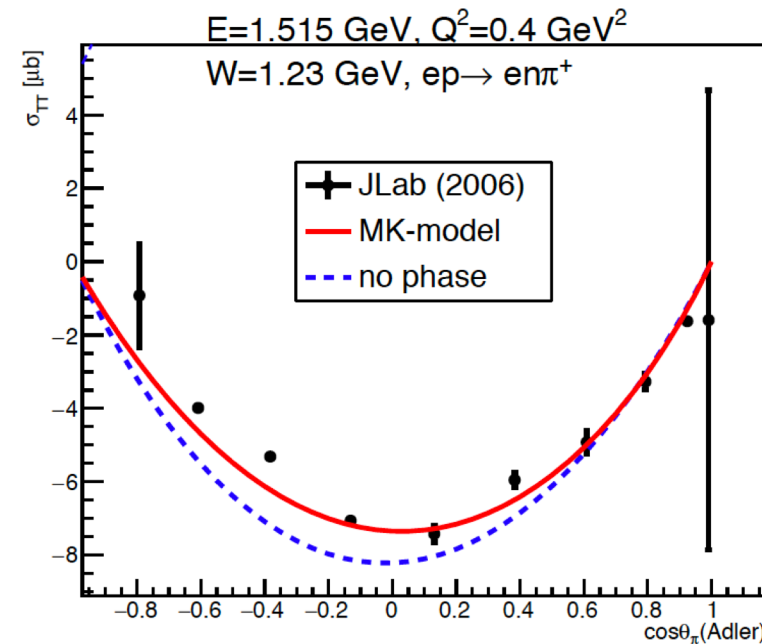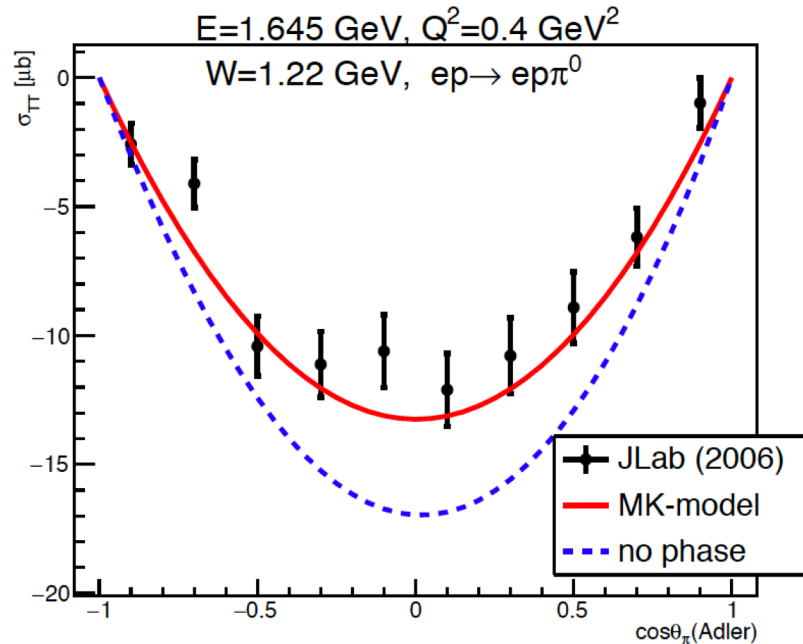# MK model comparison with J-lab exclusive data
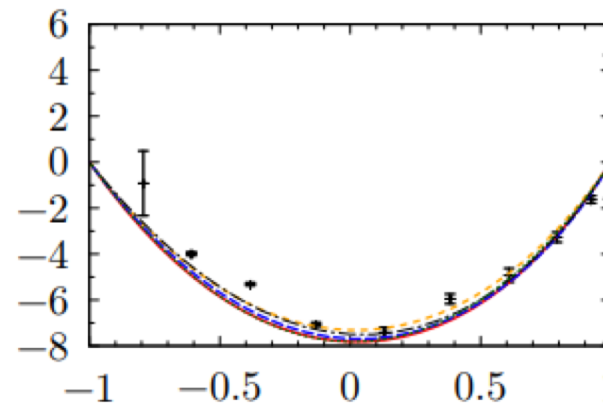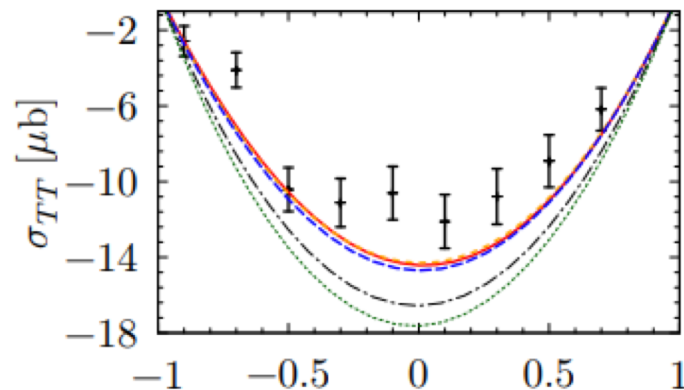

$$\sigma_T + \varepsilon\sigma_L$$

Phys. Rev D.98, 073001 (2018)

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MK model comparison with J-lab exclusive data

$$\sigma_{TT}$$

Phys. Rev D.98, 073001 (2018)

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MK model comparison with J-lab exclusive data

E=1.645 GeV, Q^2=0.4 GeV^2
W=1.30 GeV, $ep \to ep\pi^0$

E=1.515 GeV, Q^2=0.4 GeV^2
W=1.29 GeV, $ep \to en\pi^+$

σ_{TT}

Jlab data
MK model
no phase

JLab (2006)
MK-model
no phase

DCC — SL — HNV
HNV1 — HNV2

Phys. Rev D.98, 073001 (2018)

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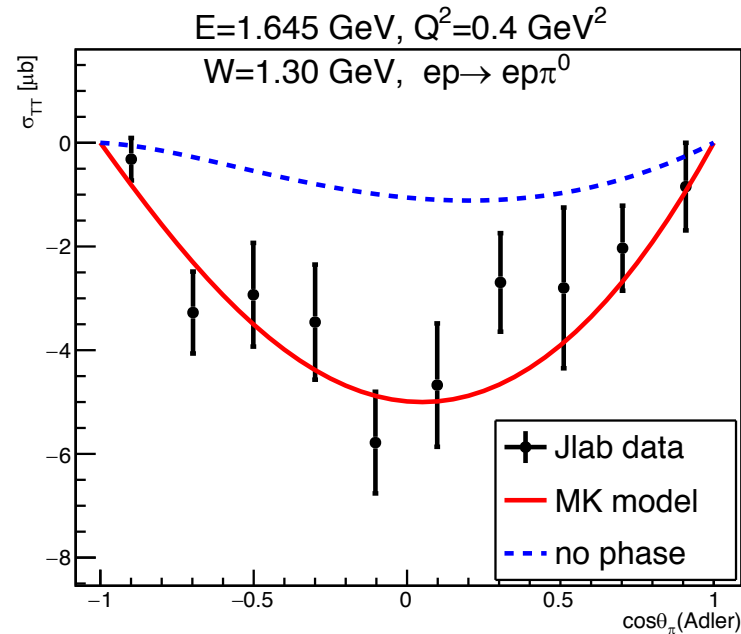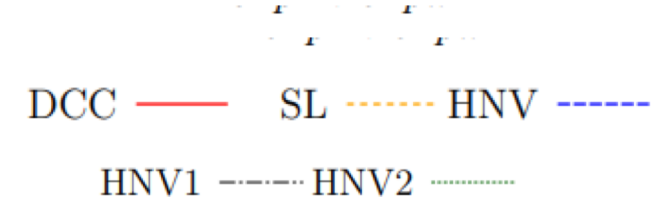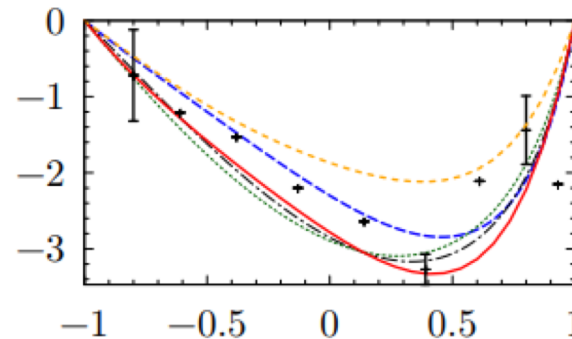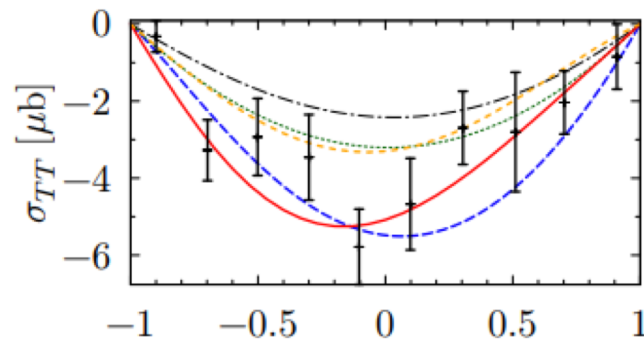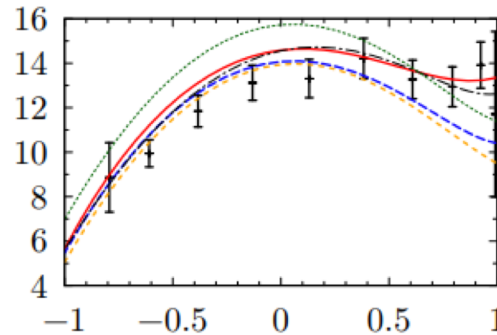# MK model comparison with J-lab exclusive data

E=1.645 GeV, Q^2=0.4 GeV^2
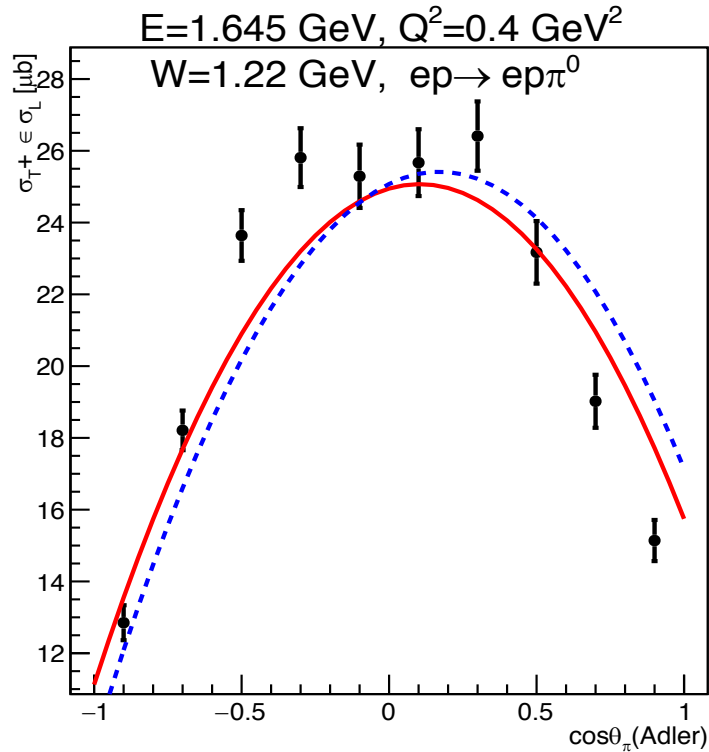W=1.22 GeV, ep→ epπ^0

E=1.515 GeV, Q^2=0.4 GeV^2
W=1.23 GeV, ep→ enπ^+

$\sigma_T + \varepsilon\sigma_L$

- JLab (2006)
- MK-model
- no phase

DCC —— SL ----- HNV ----

HNV1 ---·--- HNV2 ·········

Phys. Rev D.98, 073001 (2018)

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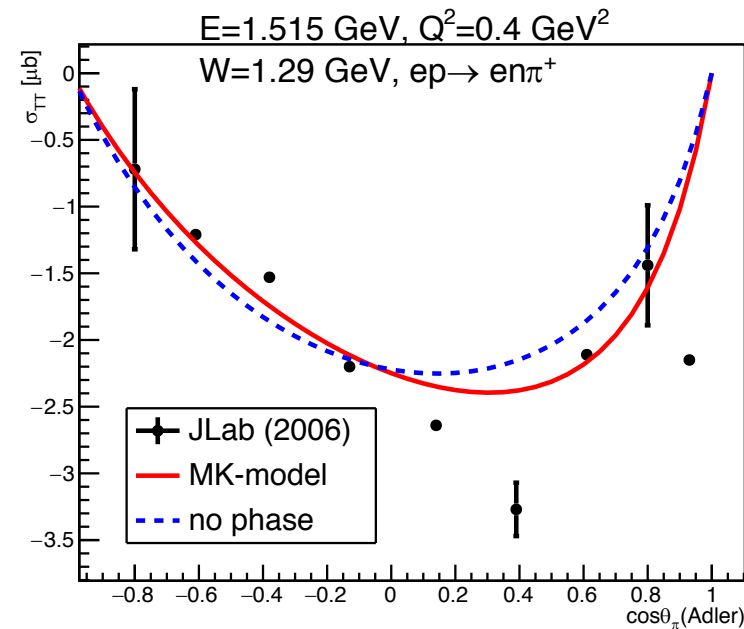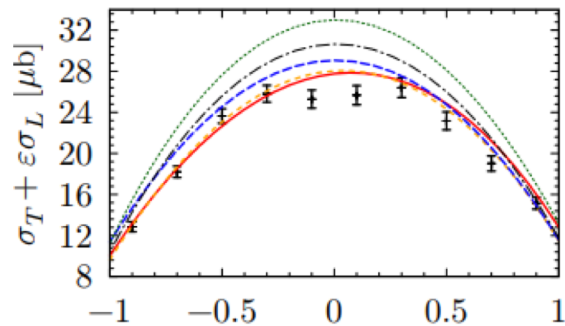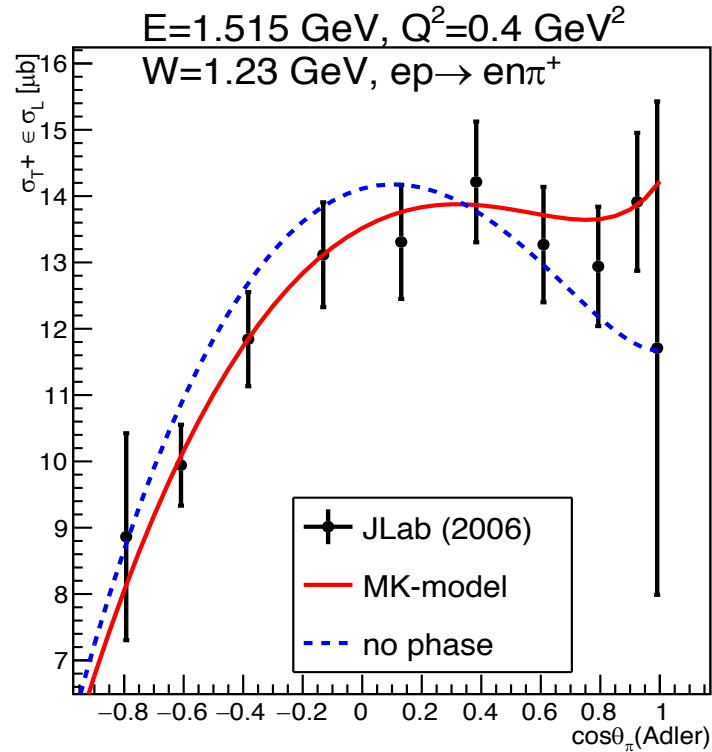
MK-model improvement
Vector part

1. Updating axial form-factor

- MK model used to have two form-factors (RS & GS)
- GS form-factor used to be the default form-factor.
- GSK is the updated form-factor for MK model. (see next slide)

2. fitting phases between resonance and non-resonance amplitudes with electron scattering data.

3. Fitting The axial form-factor at $Q^2=0$ (only $C^5_A(0)$)

Graczyk-Sobczyk axial form-factor

- They equivalent the RS model with Lalakiluch et al model (Rarita-Schwinger formalism)

$$\widetilde{G}_A^{RS,+3,+1}(W,Q^2) = \frac{\sqrt{3}}{2}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[1-\frac{W^2-Q^2-M^2}{8M^2}\right]C_5^A(Q^2), \quad \textbf{GS}$$

$$\widetilde{G}_A^{RS,+0}(W,Q^2) = \frac{\sqrt{3}}{2}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[\frac{W^2-Q^2-M^2}{2W(W-M)}+\frac{WQ^2}{4M^2(W-M)}\right]C_5^A(Q^2) \quad \textbf{GSK}$$

$$\widetilde{G}_A^{RS,+3,+1}(W=M_\Delta,Q^2=0) \quad \approx \quad \widetilde{G}_A^{RS,+0}(W=M_\Delta,Q^2=0)$$

- In order to be able to get an agreement with both sets of data they choose GS form factor.

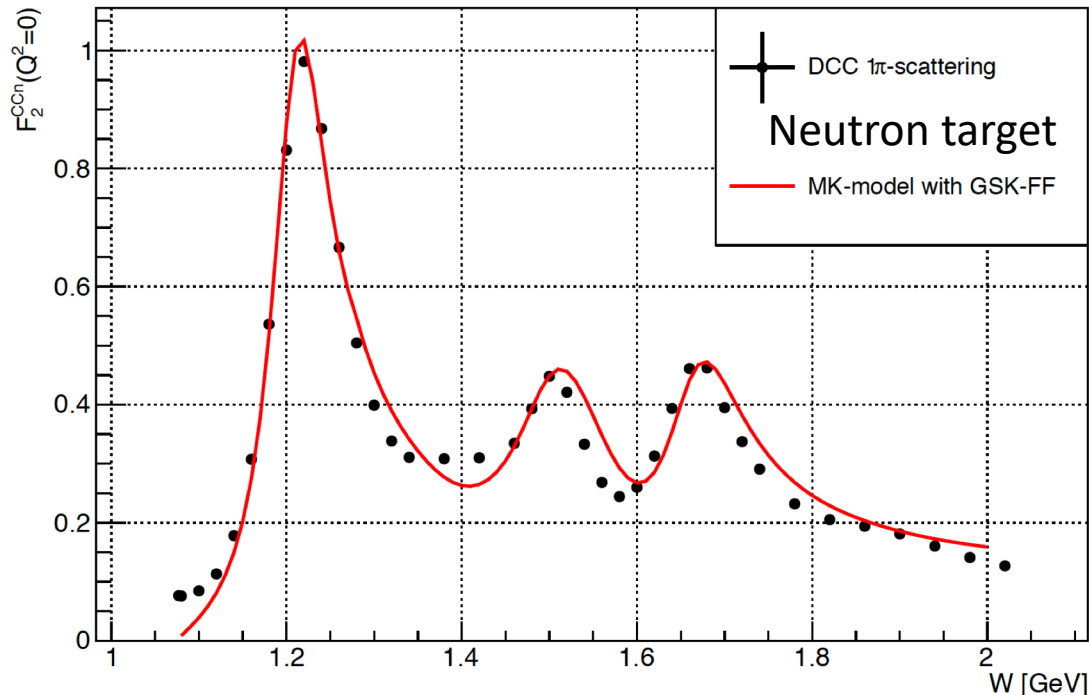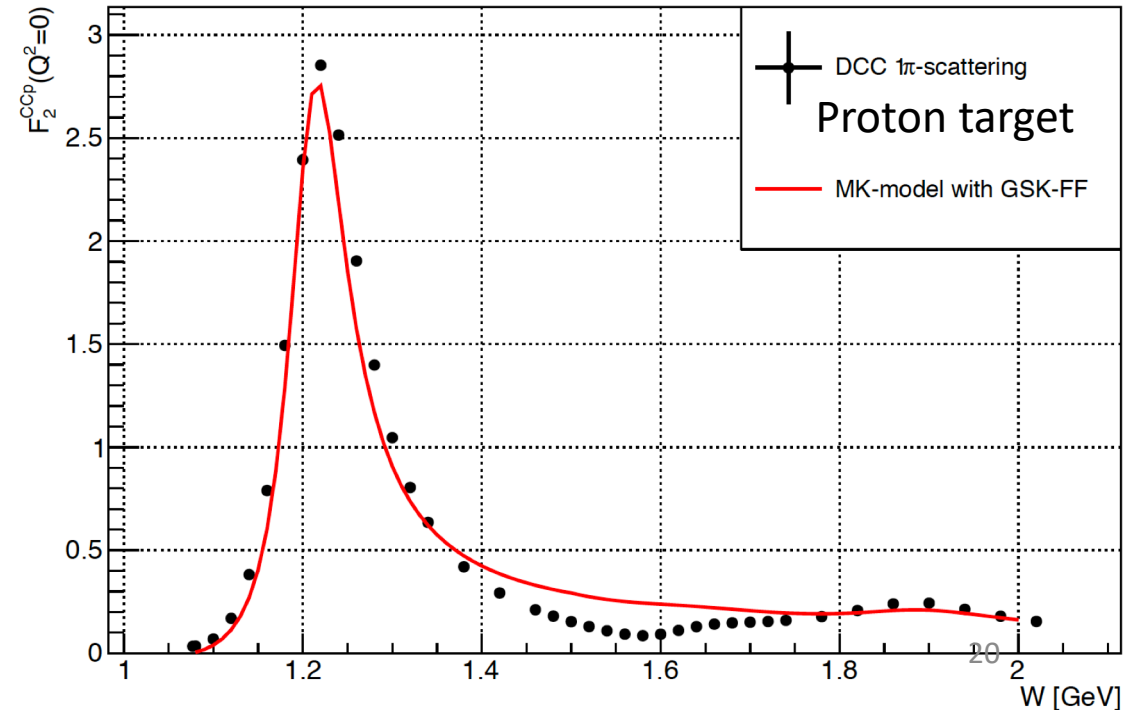- MK model with GSK form-factor has better agreement with data

MK-model improvement (Axial part)

At this particular kinematics, the axial part of neutrino cross section is related to the πN cross section through the PCAC relation.

Cross section at $Q^2=0$ and $m_\mu=0$

$$\frac{d\sigma^{CC}}{dE_l d\Omega_l} = \frac{G_F^2 V_{ud}^2}{2\pi^2} \frac{E'^2}{E-E'} F_2,$$

PCAC

$$F_2 = \frac{2f_\pi^2}{\pi} \sigma_{tot}(\pi+N)$$

$Q^2=0$

$Q^2=0$

Conclusion

- MK-model (single pion cross section model) consists of resonant and non-resonant interactions, **including the interference effects**.

- MK-model has a good agreement with hydrogen/deuterium target (neutrino/electron/pion) data after updating the vector form factor.

- Updated MK-model can have very different prediction for some channels.

- MK model implementation in NEUT consists of 14 (channels) functions which each function get E, Q^2, W, θ_π, ϕ_π and returns to differential cross section ($d\sigma/dW \, dQ^2 d\Omega_\pi$)value for individual channels.

- GENIE implementation is not as easy as the NEUT implementation!

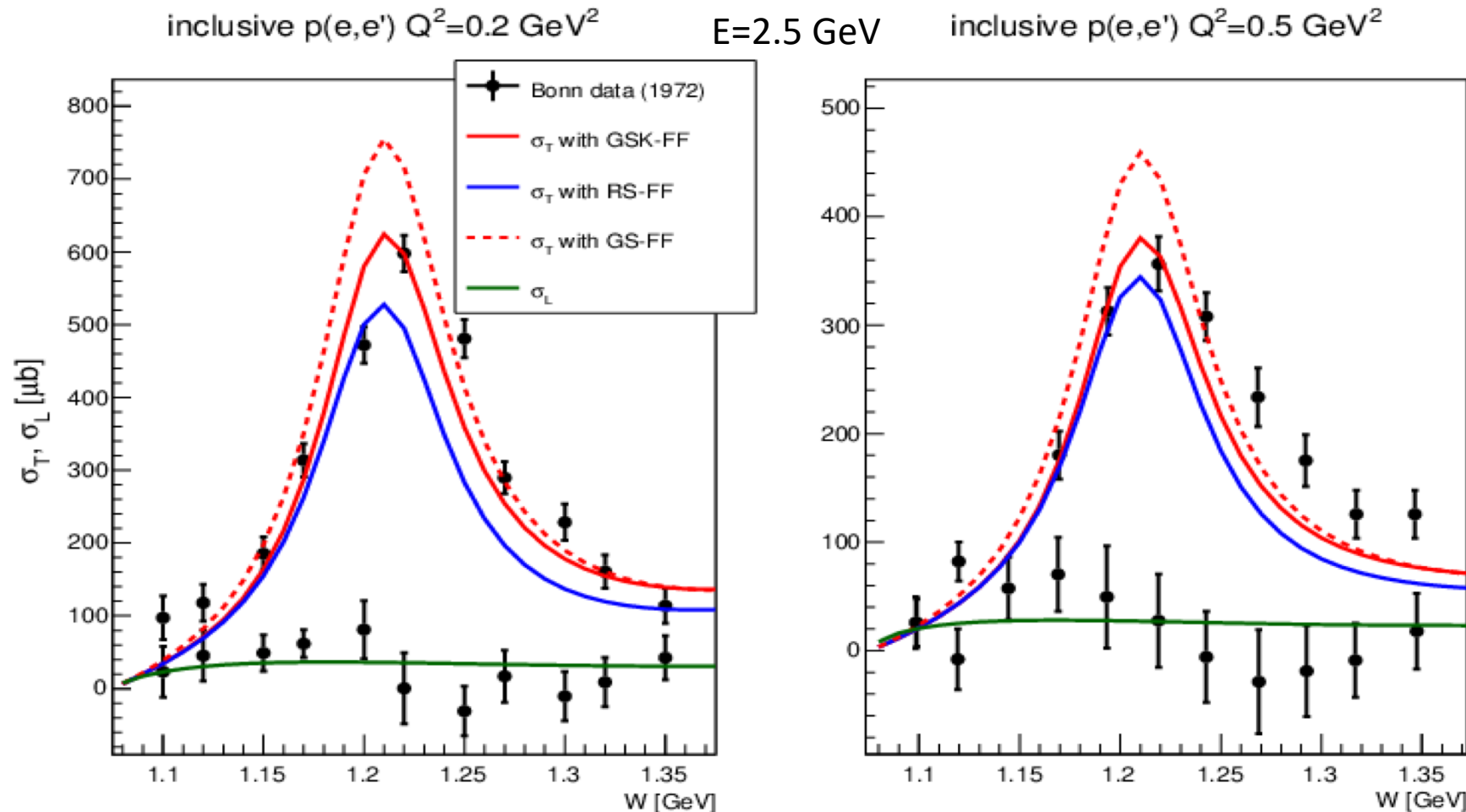# Backup

Bonn Inclusive *ep* Xsec

$$\frac{d^3\sigma_{ep\to e'X}}{dE_{e'}\,d\Omega_{e'}} = \Gamma_\gamma[\sigma_T(W,Q^2) + \epsilon\sigma_L(W,Q^2)]$$

virtual photon flux factor
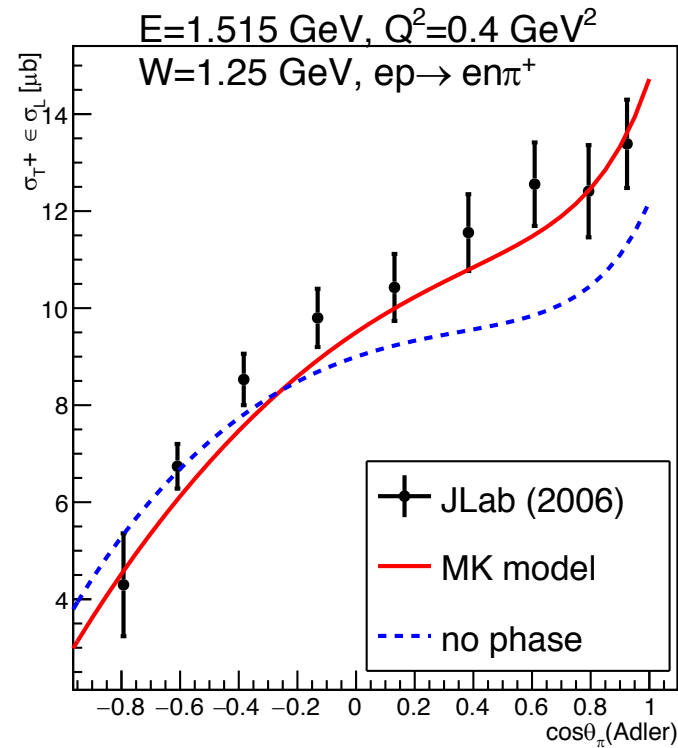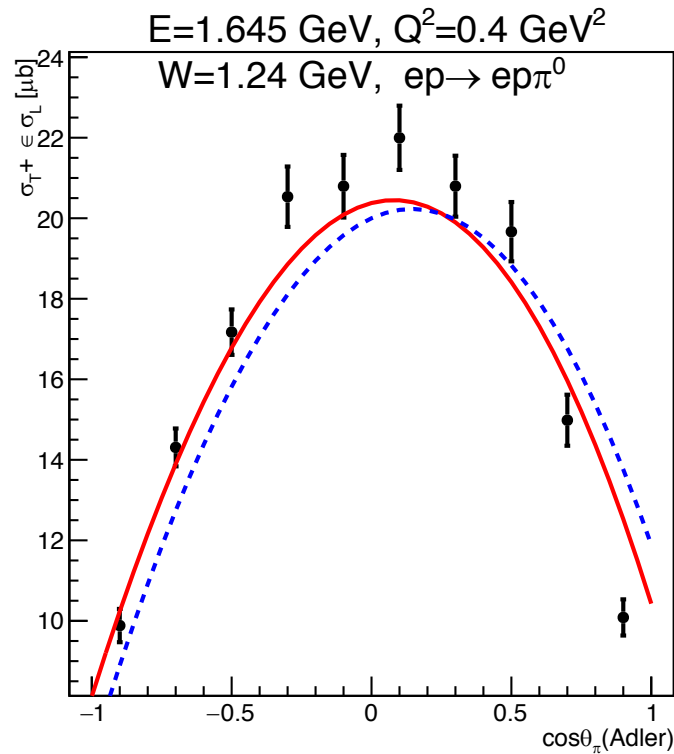
$$\Gamma \equiv \frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{(W^2-m_p^2)}{2m_pQ^2}\frac{1}{1-\epsilon}$$

$$\epsilon \equiv \left(1 + 2\frac{|\mathbf{q}|^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1},$$

E=2.5 GeV

inclusive p(e,e') Q^2=0.2 GeV^2

- Bonn data (1972)
- σ$_T$ with GSK-FF
- σ$_T$ with RS-FF
- σ$_T$ with GS-FF
- σ$_L$

inclusive p(e,e') Q^2=0.5 GeV^2

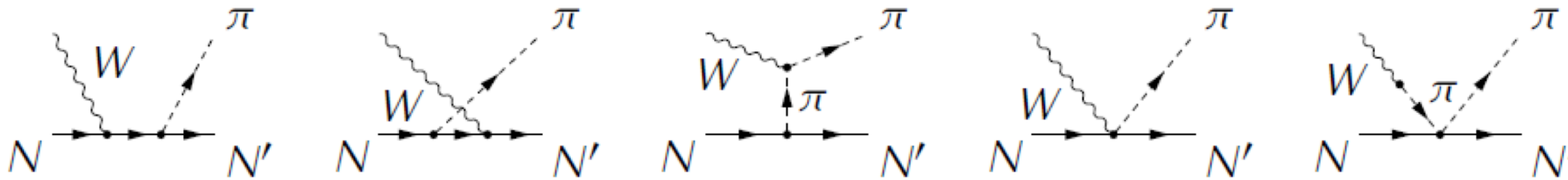No adjustable parameter in vector form-factors

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MK model comparison with J-lab data

$$\sigma_T + \varepsilon\sigma_L$$

non-resonant background.

is defined by a set of diagrams determined by HNV model based on non-linear sigma model

**Helicity amplitudes of above diagrams are calculated
in the Adler frame.**

- the model is based on chiral symmetry and it is not reliable at high energy and high W.

- Smooth transition to higher W (1.4 GeV<W<1.6 GeV)

- The nonresonant background has no contribution at W>1.6 GeV

Dynamical coupled-channels (DCC) model

DCC analysis of meson production data

- **Fully combined** analysis of $\gamma N, \pi N \Rightarrow \pi N, \eta N, K\Lambda, K\Sigma$ data

  ~ 27,000 data points are fitted

- In first analysis of the pion- and photon-induced meson production reactions, we have already constructed a DCC model for the strong interaction and the electromagnetic current of the proton at $Q^2 = 0$.

- More than 440 parameters are determined to fit the obtained vector form factors.

$$F_{NN^*}^V(Q^2) \sim \sum_{n=0}^{N} c_n^N (Q^2)^n$$

- All the other (406) parameters such as resonance parameters (masses & decay widths) and relative phases between resonant and nonresonant amplitudes have been extracted from the DCC model.

Vector and axial-vector currents

q.V (Q^2=0)=0   Conservation of Vector current (CVC)

q.A(Q^2=0) $\propto m_\pi^2 \neq 0$ axial current is not conserved. But it is partially conserved (PCAC) when $m_\pi \rightarrow 0$

→ Guiding principle to derive the axial current : PCAC relation with πN reaction amplitude

$$<X|q \cdot A(Q^2 \sim 0)|N> \sim i f_\pi <X|T|\pi N>$$