SuSAv2 in GENIE

arXiv:1905.08556

***Stephen Dolan**, Guillermo Megias and Sara Bolognesi*

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Overview

Based on: arXiv:1905.08556

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- Based on implementations of the Valencia 2p2h (NEUT/GENIE)

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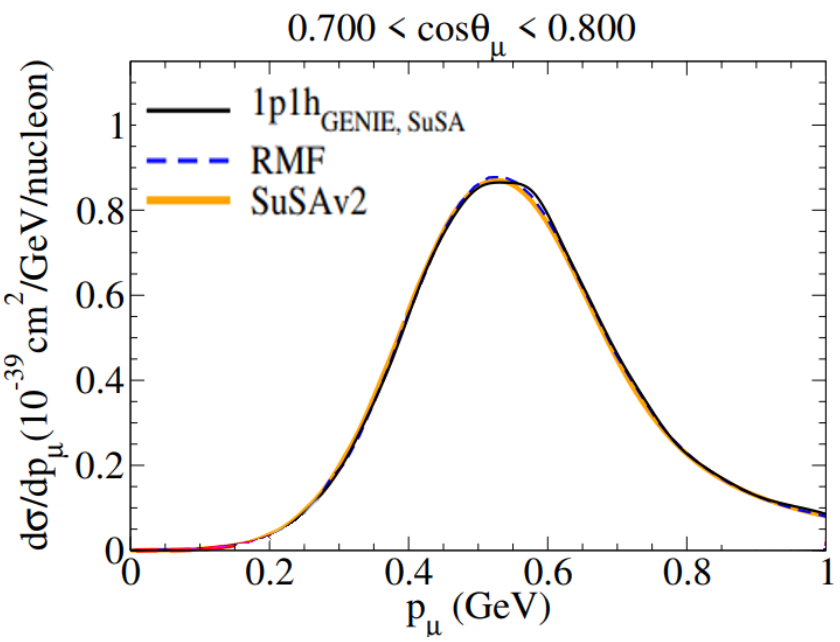

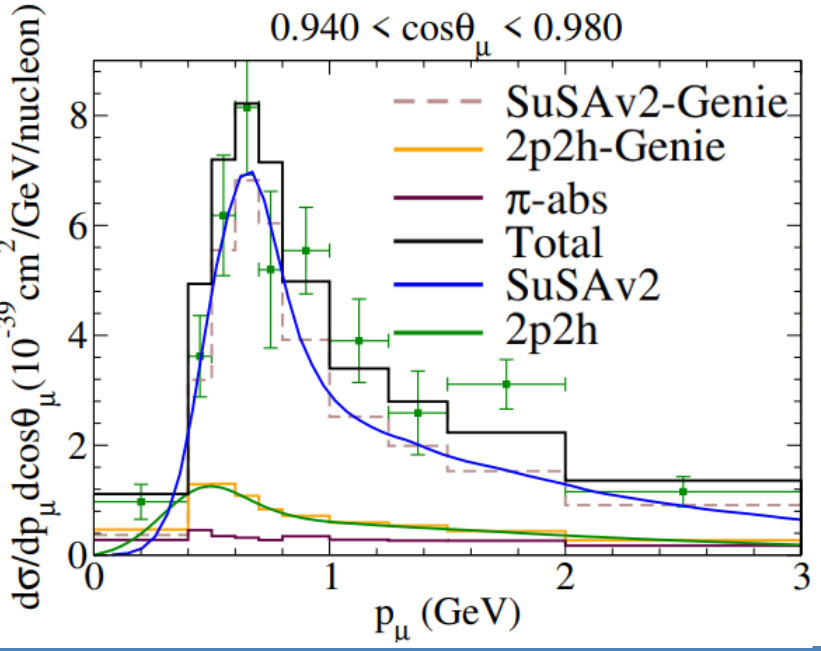

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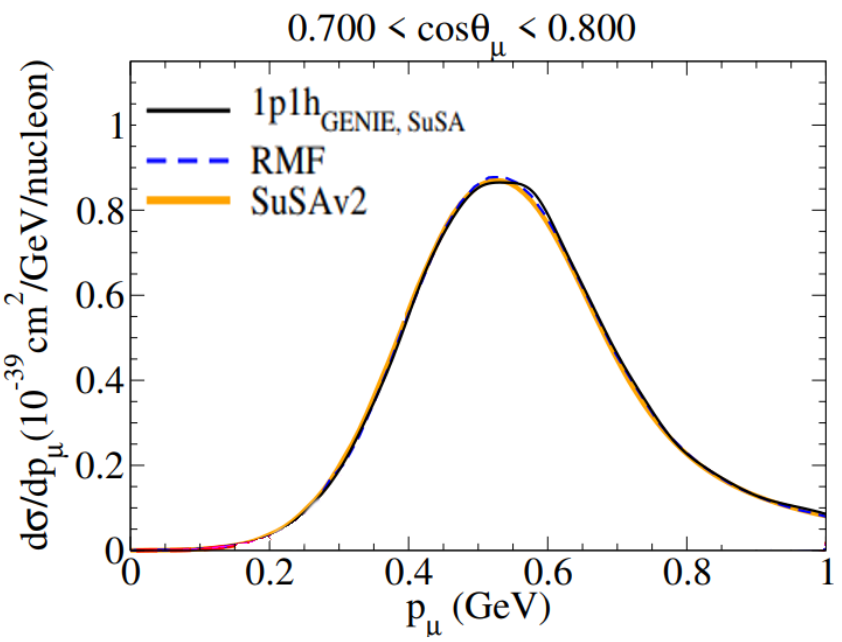

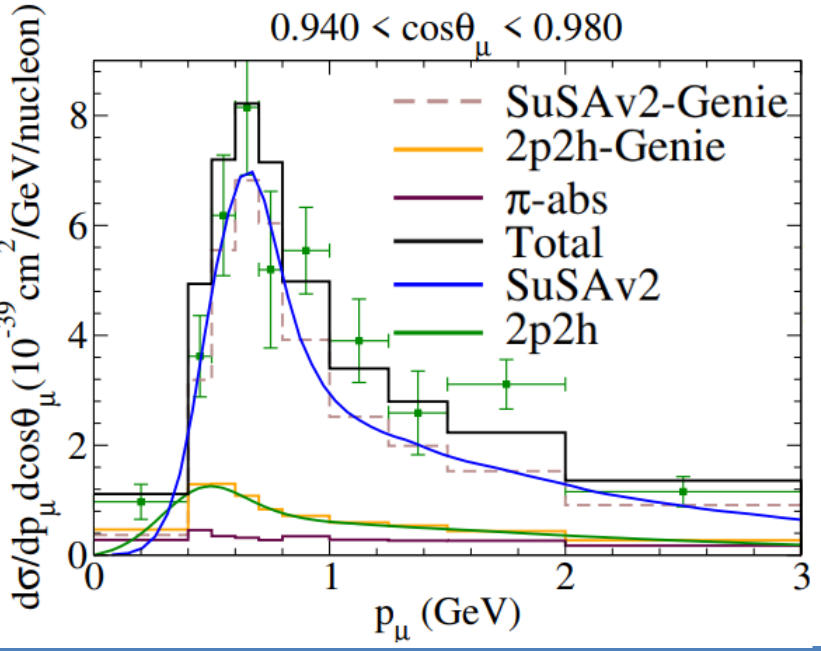

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This talk:

- ~~Introduction to the model~~
- GENIEv3 implementation
- Validations
- Technical details and open questions
- Limitations and future improvements

Implementation

Like the Valencia 2p2h model, SuSAv2 is able to predict only the outgoing lepton kinematics. The double differential cross section can be written:

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  - Using Steven's great new hadron tensor framework with a custom Rosenbluth decomposition based xsec evaluation method
  - Interpolate between points to evaluate a specific q0,q3
  - 2p2h tensors are split into np and pp pairs
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  - Hadron tensors will be provided for a few select targets (just C so far, others to come). Can accurately scale to other nuclei.
- GENIE implements the 2p2h hadron tensor from a full microscopic calculation, but this is not what is often used by the SuSA group (who use a parameterization)
  - GENIE's implementation is slightly more true to the full calculation (most of the time)

New feature: $\bar{\nu}$-H

- SuSAv2 1p1h cannot calculate antineutrino-nucleon ($\bar{\nu}$-H) elastic scattering. SuSAv2 relies on having two dimensional hadron tensors and xsecs, but the elastic cross section is only one dimensional.

- ***New*** *(thanks to Steven G)*: when using SuSAv2 1p1h and asking for an elastic cross section use the Llewellyn Smith (LS) model

Hadron kinematics: The ugly truth

Xsec calculation:

- Perform inclusive calculation (q_0, q_3) using SuSAv2 hadron tensor

Hadronic side: more complicated …
Bold/*italics* **bits represent very/***slightly* **questionable physics**, but this is no worse than in most other model implementations.

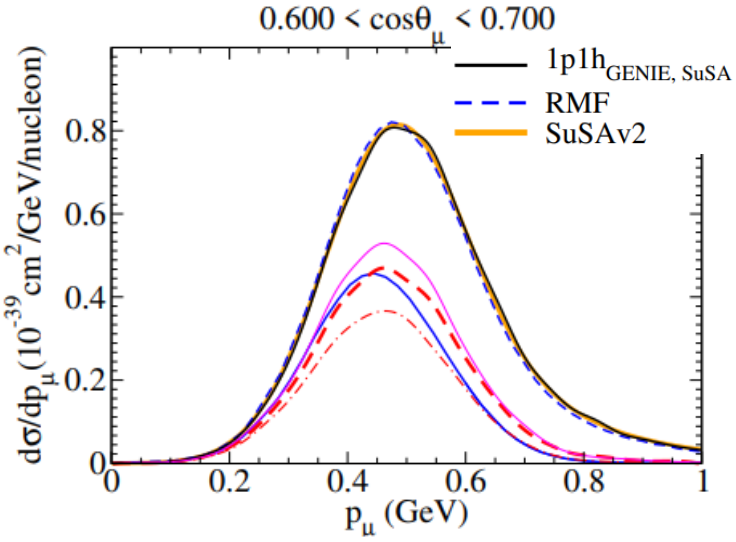
- Draw target nucleon from chosen nuclear model **irrespective of q_0, q_3**
- Get removal energy from RMF-like treatment in 1p1h or GENIE's nuclear model for 2p2h, *re-throw from nuclear model if nucleon is Pauli blocked*
- Transfer all of ω, q to nucleon, **none to remnant**
- Subtract removal energy, **put proton on-shell with adjustment of p** (only needed for 1p1h) **then conserve momentum by adjusting remnant kinematics**
- *Do FSI cascade and rest of interaction using standard GENIE methods*

Validations

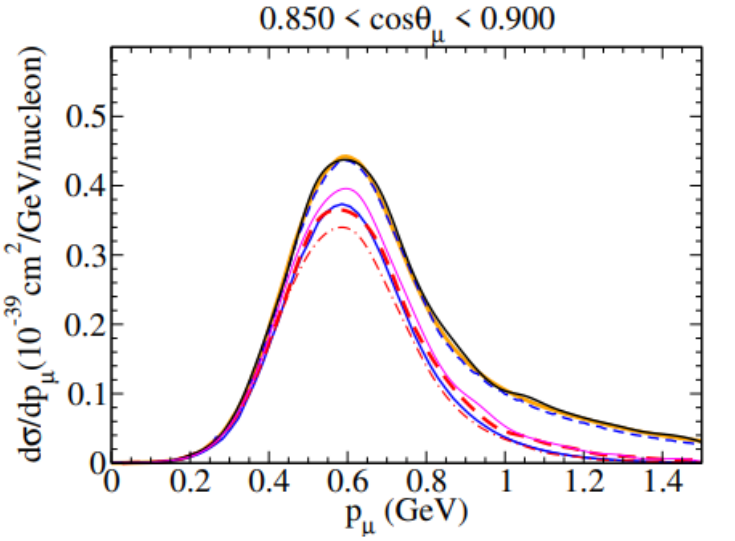
Point checks

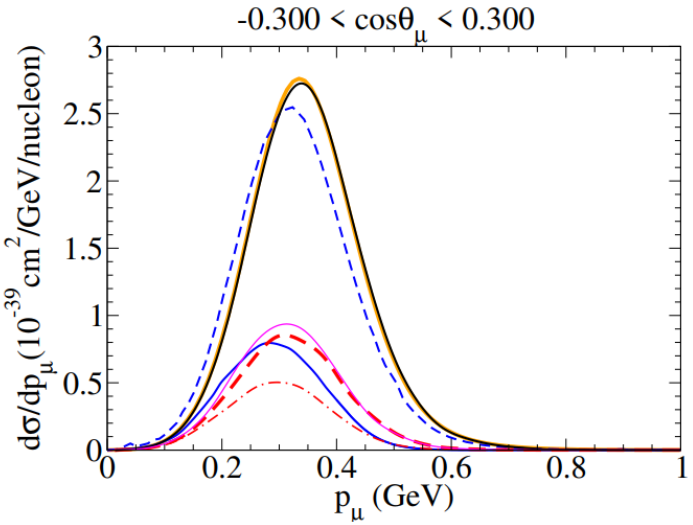
- Checked a few dozen single points in $E_\nu, p_\ell, \cos\theta_\ell$ for 1p1h, 2p2h, 2p2h with only np pairs and for neutrino and electron scattering
- At each point calculated each of the hadron tensor elements and the differential cross section
- Calculated the points in GENIE directly from the hadron tensors (no use of the QE event generator classes) and compared to SuSAv2 calculations
- Always correct within ~3% (mostly better). Differences are understood to come from interpolation and integration method differences.
- Separately check that the xsec calculated after running the event generator is the same as the one in the input model (additional hadronic effects are not allowed to alter the inclusive model predictions!)

1p1h differential cross section

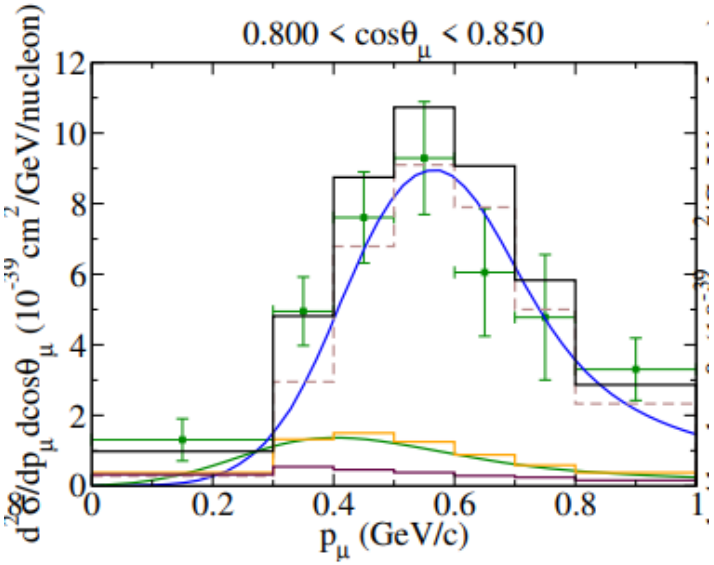


- Comparison of the 1p1h double differential cross section for T2K flux
- **Validation: the black solid line and green thick line are the same. Ignore the rest**
- More plots are in arXiv:1905.08556 Fig. 12

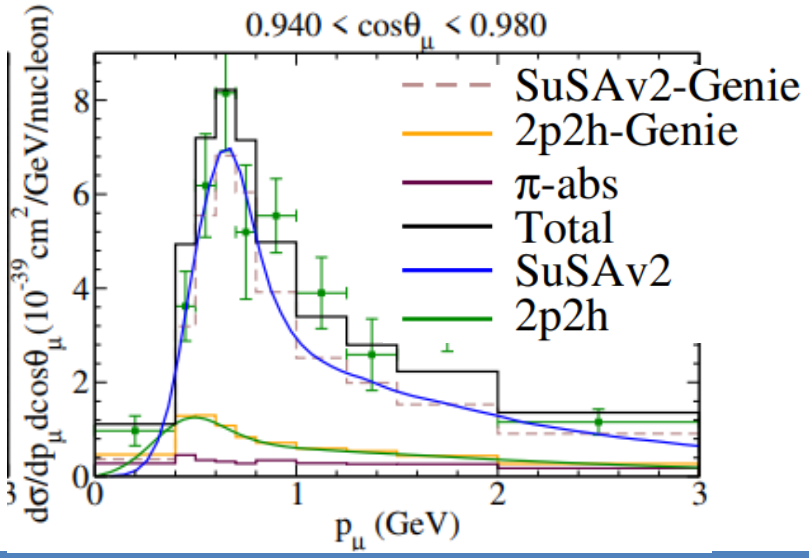


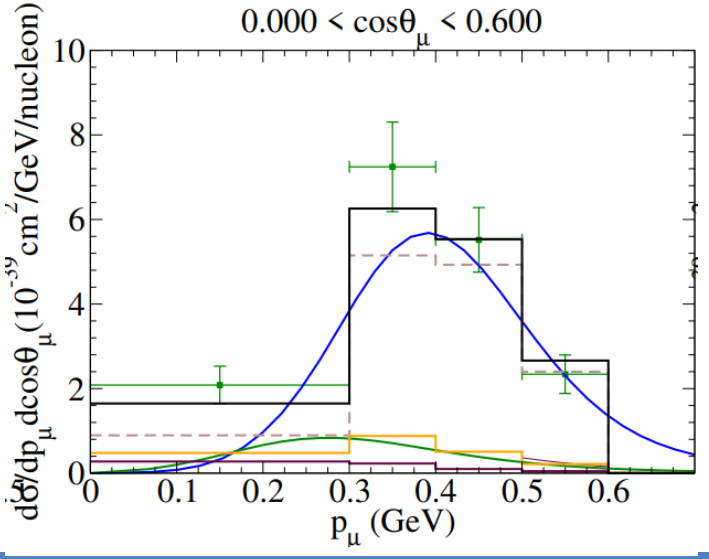


CC0π differential cross section



- Comparison of the CC0π double differential cross section for T2K flux
- **Validation: the histograms (GENIE) are the same as the curves (model)**
- More plots are in arXiv:1905.08556 Fig. 8





Further details

More technical details

- **New event generator:** Evaluation of 1p1h cross sections in $p_\ell, \cos\theta_\ell$ requires a new QE event generator – looks similar to the 2p2h one

- **Hadron kinematics:** SuSAv2 only provides an inclusive calculation – calculation of hadron kinematic is ad-hoc. I added some parameters so the users can change some of the default assumptions I make to generate these.

- **Binding energy:** 1p1h hadron kinematic calculation includes an optional q_3 dependent binding energy inspired by RMF.

- **Accept/Reject max xsec:** The 1p1h max xsec calculation is based on existing GENIE methods and uses caching but currently seems fairly inefficient and requires a large safety factor (5). Things could be improved here.

- **Scaling beyond Carbon:** Scaling to other nuclear targets is done by applying the scaling rules: but I use GENIE's table of k_F which is different that SuSA's. I also apply a shift to the tensor to account for different targets binding energy cut offs.

Validations ongoing – e,e'

- Exactly the same implementation is also used to implement SuSAv2 e,e' scattering (for 1p1h and 2p2h)

- However there seems to be a bug or some missing factor in the tensor tables. Guillermo and I will work to solve this but it shouldn't hold up the release of the neutrino scattering model.

Limitations and future work

Current status

- The SuSAv2 1p1h and 2p2h implementations are ready and validated
- Exactly reproduces *inclusive* input model predictions, which is all the input model provides
- The current version of the code is ready and will make a useful and complementary addition to GENIE
- **How will the model be useful?**
  - Additional model to compare to xsec results
  - Can motivate mock-data studies or systematics in oscillation analyses
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  - A consistent 1p1h+2p2h model that can be compared to e,e' and neutrino data
- **What can't it do?**
  - Reliably predict hadron kinematics (like almost all models …)
  - Act as a reweightable input to an analysis

Limitations

- SuSAv2 is an inclusive model, it can only predict the outgoing lepton kinematics, anything we do with hadrons is approximate at best

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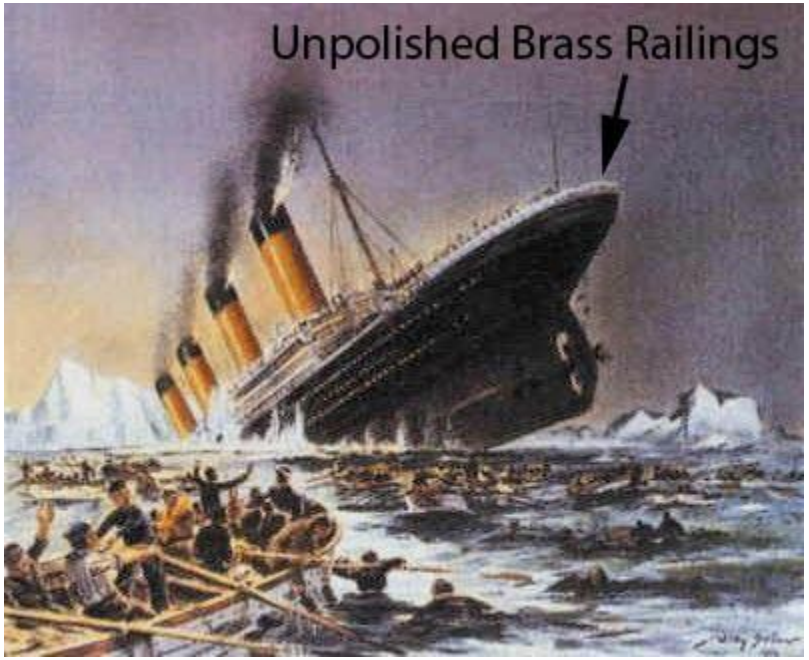
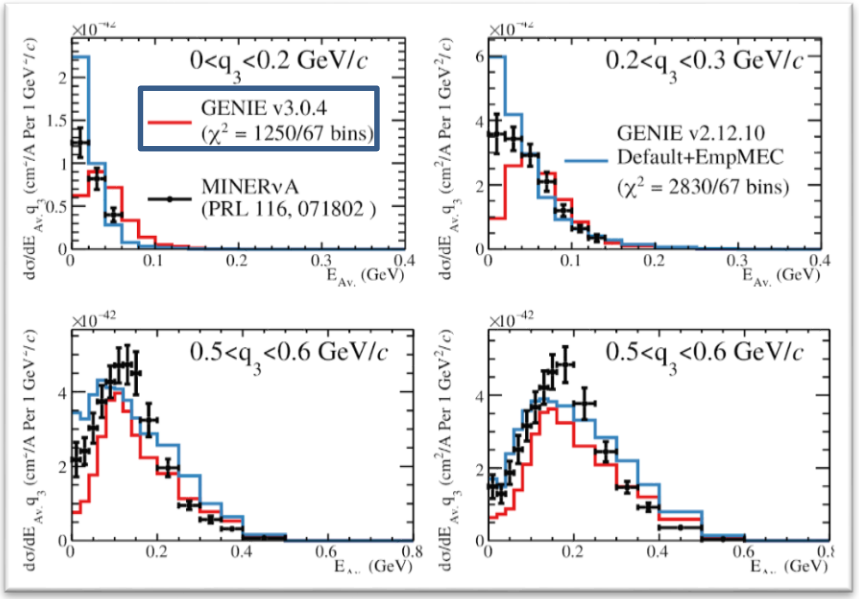
Lots of effective treatments we use to improve things (see backups). But to me this is just "polishing the brass on the Titanic".



Beyond SuSAv2

- If we want to really describe hadron kinematics better than **this** …



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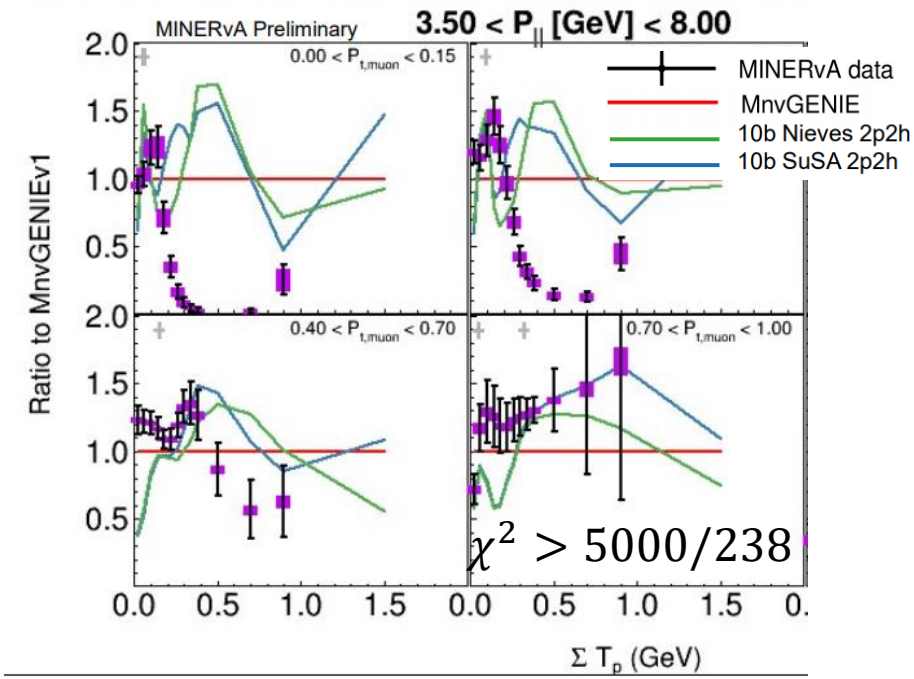

Daniel Ruterbories, JETP, 25/10/19

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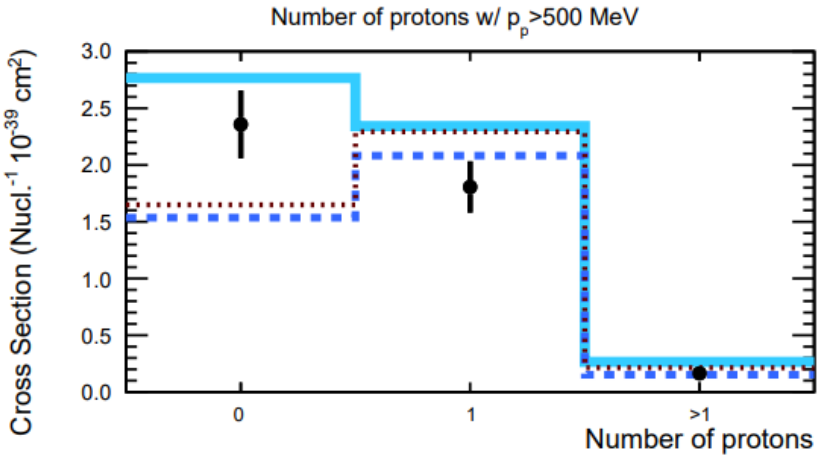

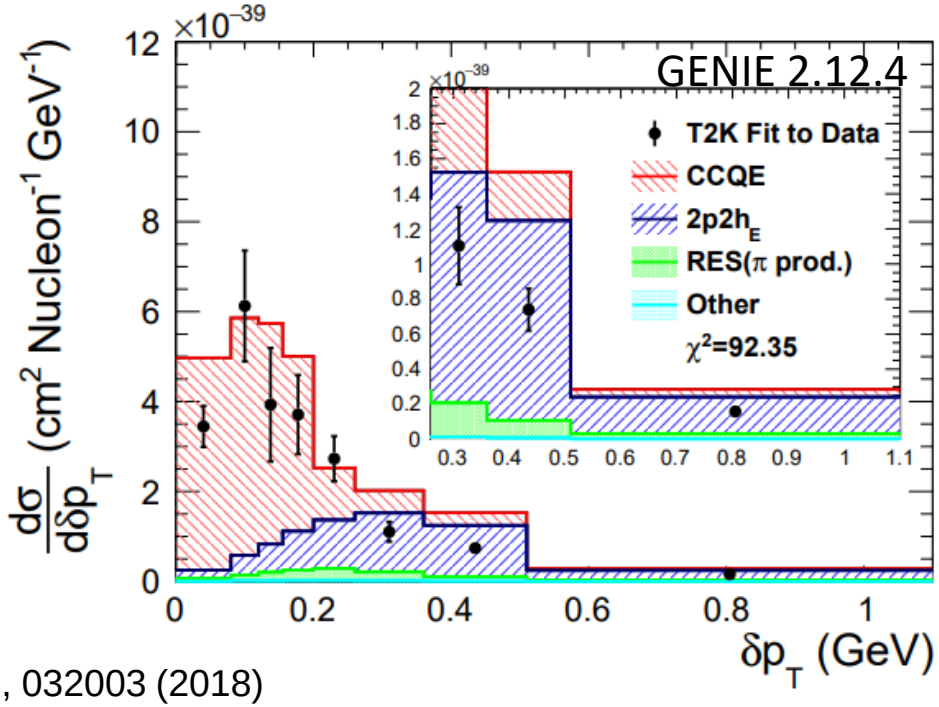

Phys. Rev. D 98, 032003 (2018)

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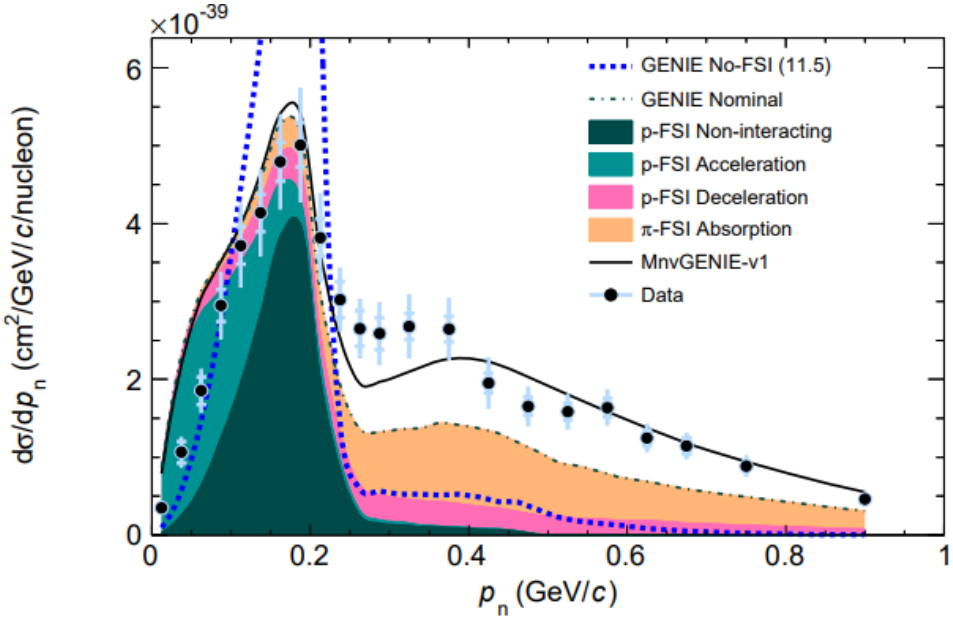
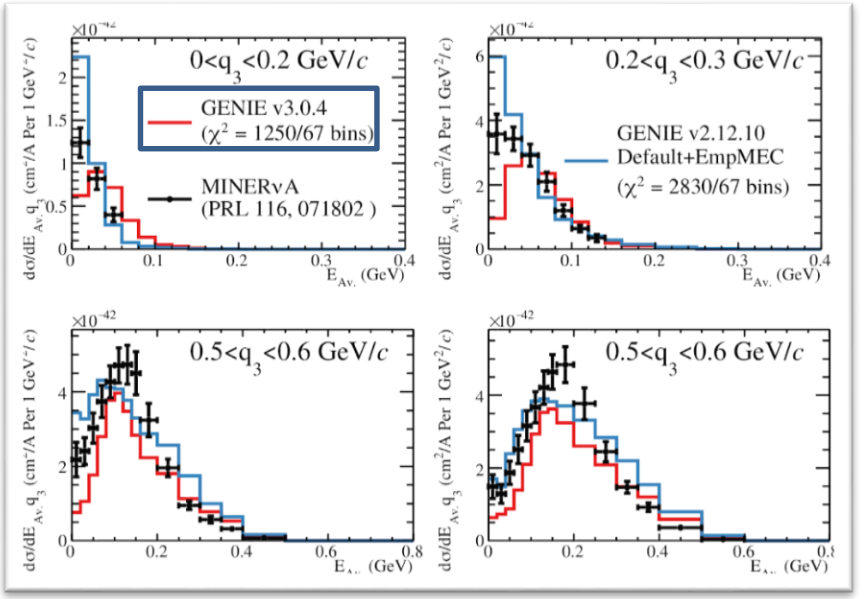


Phys. Rev. Lett. 121, 022504 (2018)

Beyond SuSAv2

- If we want to really describe hadron kinematics better than **this** …
- … then we need to start using exclusive models with predictive power in electron scattering.
- SuSAv2 is effectively an inclusive and fast version of such a model: RMF
- Implementation of RMF (or the improved ED-RMF) would be a great step forward (and it's also available for pions)!



Next steps

Two options:

- Directly implement 5D RMF cross section
  - Most complete implementation, but slow
  - Maybe could find ways to remove/parameterise some dimensions / responses …

- If we use PWIA (not RPWIA) the cross section factorises as in SF formalisms:

$$\frac{d^6\sigma}{\mathrm{d}\omega d\boldsymbol{q} dE_m d\boldsymbol{p}_m} = KS(E_m, \boldsymbol{p}_m)\mathrm{L}_{\mu\nu}\mathrm{H}^{\mu\nu}\delta(\omega + M - E_m - E_{p'})$$

  - Then we need a 5D single nucleon hadron tensor, $H^{\mu\nu}$, (*but remember that these are bound!)* and the RMF spectral function, $S(E_m, \boldsymbol{p}_m)$.

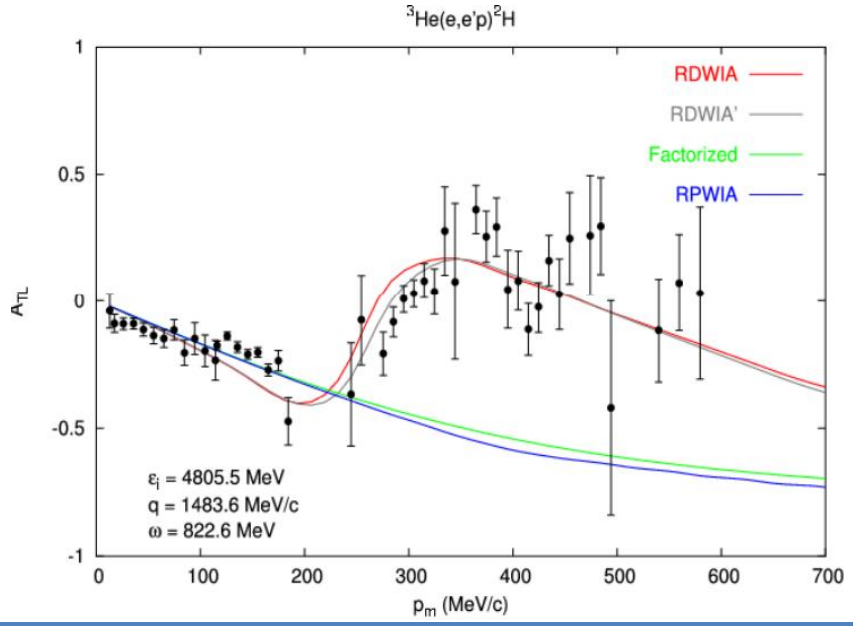
  - Much easier, but how much do we loose in this factorisation?

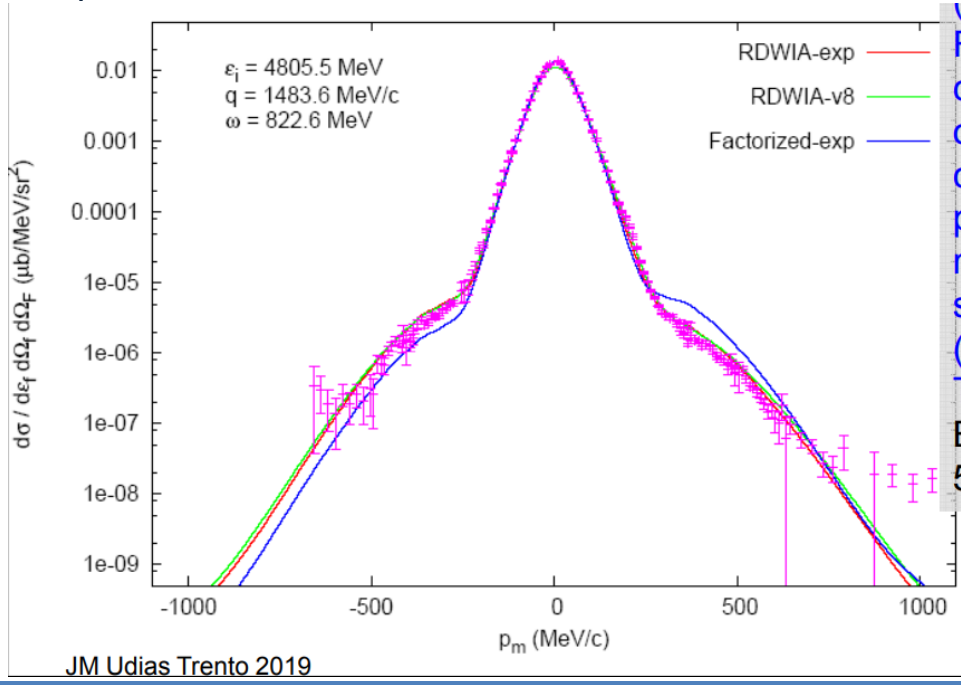
RMF SF "factorisation" approach

- Although RMF does not factorise, this approach mostly works

$$\frac{d^6\sigma}{\mathrm{d}\omega d\boldsymbol{q} dE_m d\boldsymbol{p}_m} = KS(E_m, \boldsymbol{p}_m)\mathrm{L}_{\mu\nu}\mathrm{H}^{\mu\nu}\delta(\omega + M - E_m - E_{p'})$$

- Important: the RMF SF includes the effects of FSI, correlations and the nuclear remnant – **it is not an initial nucleon momentum distribution**
- This approach is **much** easier to implement in an MC than a full 5D xsec, also retains enough control over the physics to add uncertainties
- Our current plan: start with this then compare to full calculation for neutrino relevant scenarios





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Backups

SuSA 1p1h: Scaling

Scaling is a fundamental property of lepton-nucleus scattering

SuSAv1 model: parameterize the scaling function encode nuclear dynamics

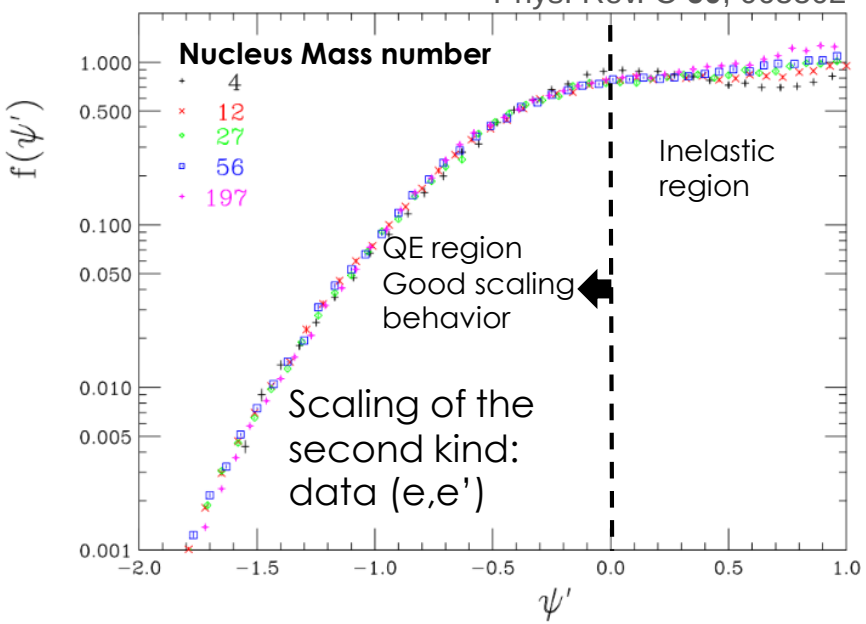
$$f(\psi) \equiv f(q,\omega) \sim \frac{\sigma_{QE}(\text{nuclear effects})}{\sigma_{\text{single nucleon}}(\text{no nuclear effects})} \longrightarrow f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega}\right)_{exp}}{\sigma_{Mott}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$

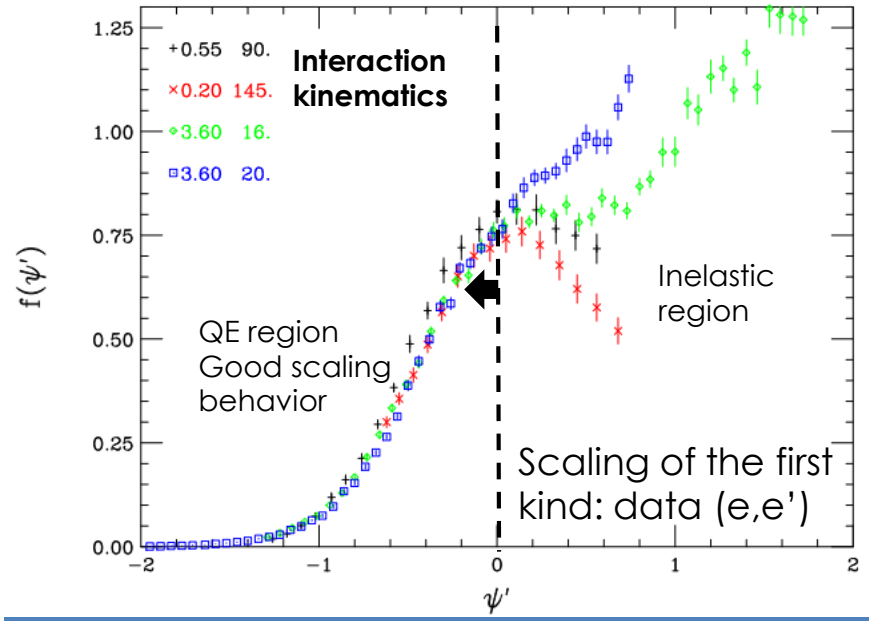
In inclusive QE scattering we can observe:

- ☆ Scaling of 1st kind (independence on q)
- ☆ Scaling of 2nd kind (independence on Z)

$\Longrightarrow$ **SuperScaling**

Phys. Rev. C **60**, 065502





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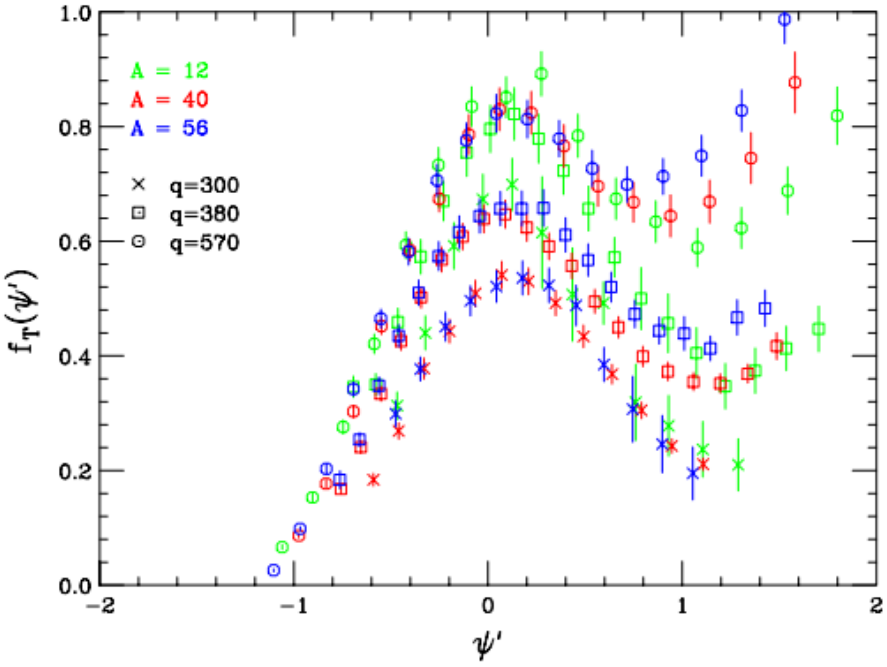
Can only extract scaling function from data for longitudinal channel: transverse has too much non-QE (which does not super-scale)



SuSA (v1): assume transverse and longitudinal scaling functions are the same and extract from data

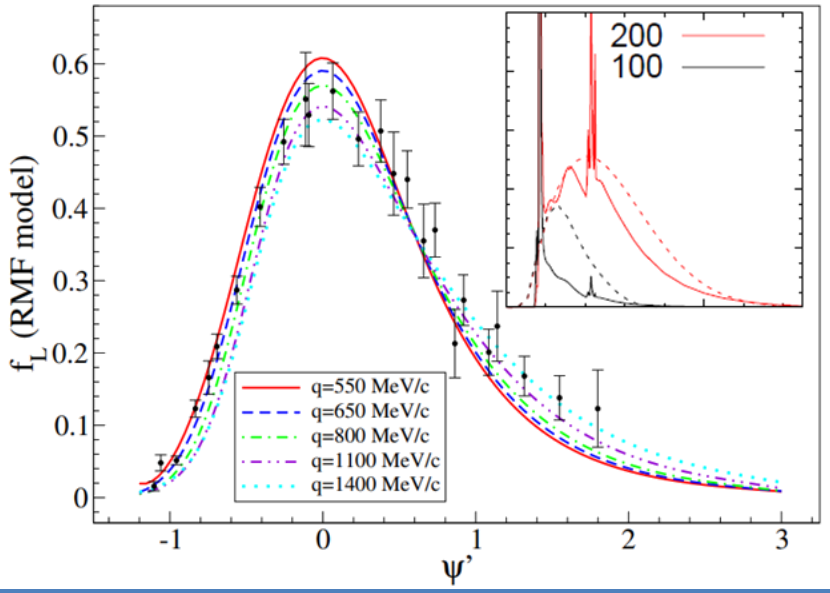
Not quite right: evidence that $f_T > f_L$ by 15%-20%

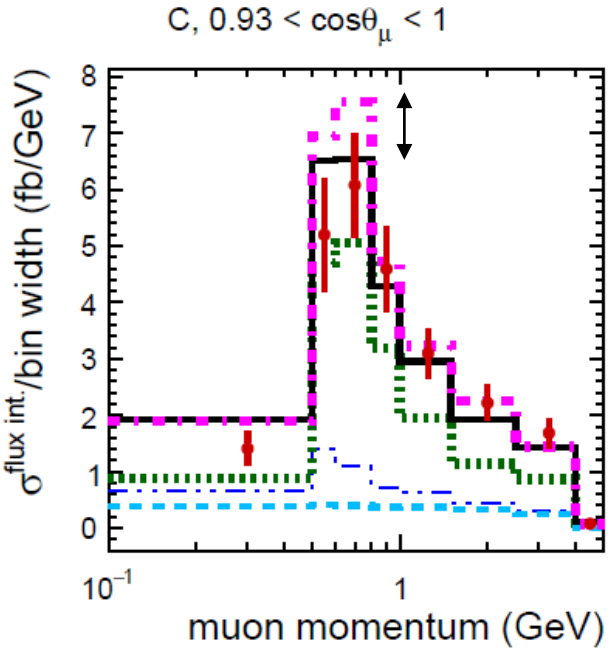
Enter SuSAv2!

SuSAv2 1p1h: use of RMF

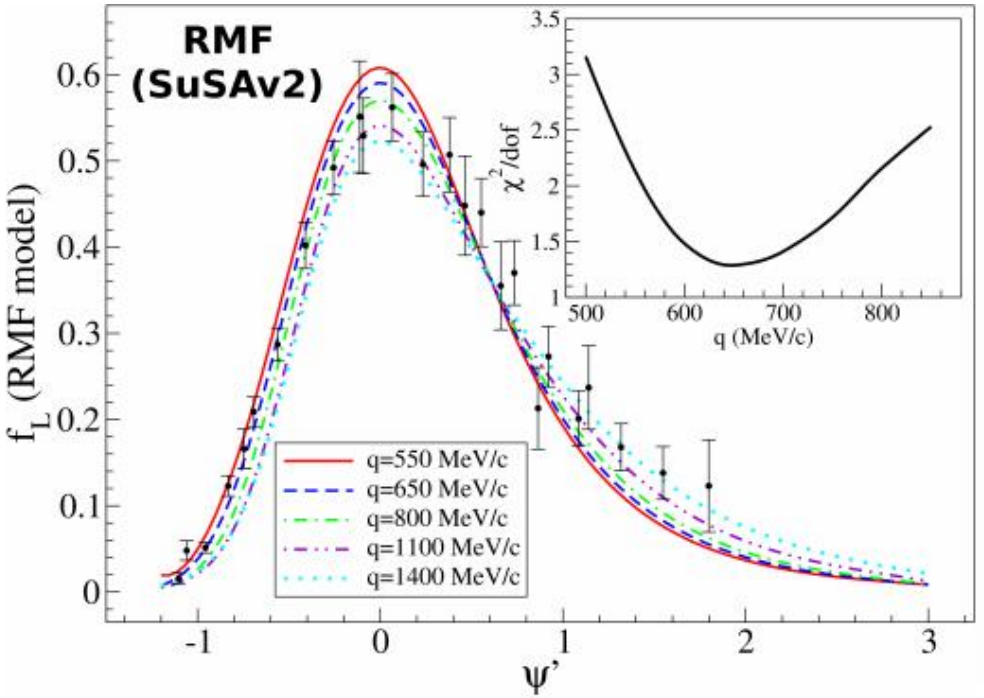
SuSAv2: build the scaling function from a microscopic model (RMF)

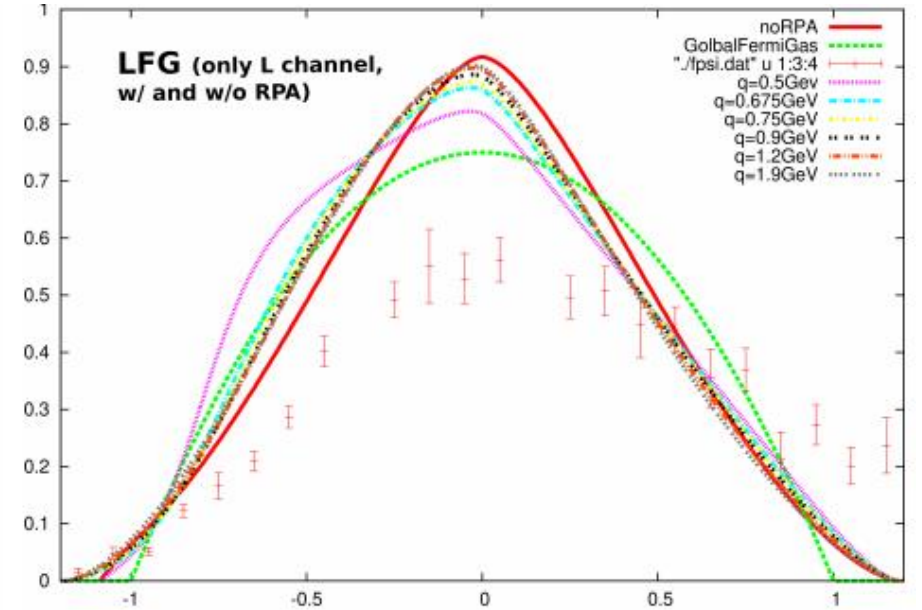
- Construct separate T and L scaling functions from microscopic model: Relativistic Mean Field (RMF) – details in backups
- RMF FSI too strong at large q: blend to RPWIA.
- Caveat: RMF slightly violates scaling of the first kind at low kinematics
  - Expect significant variations between SuSAv2 and RMF for very small ω, q_3
  - Future SuSA (v3?) will address this

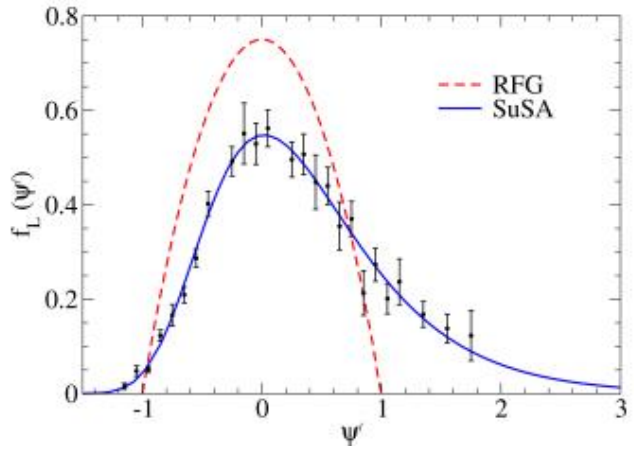


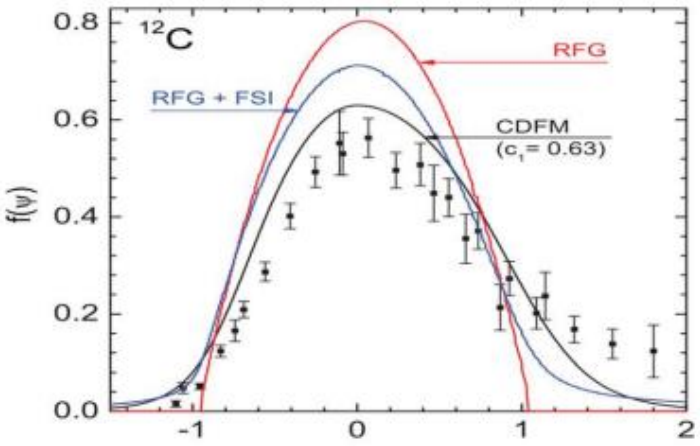


SuSAv2 – comparison to other models





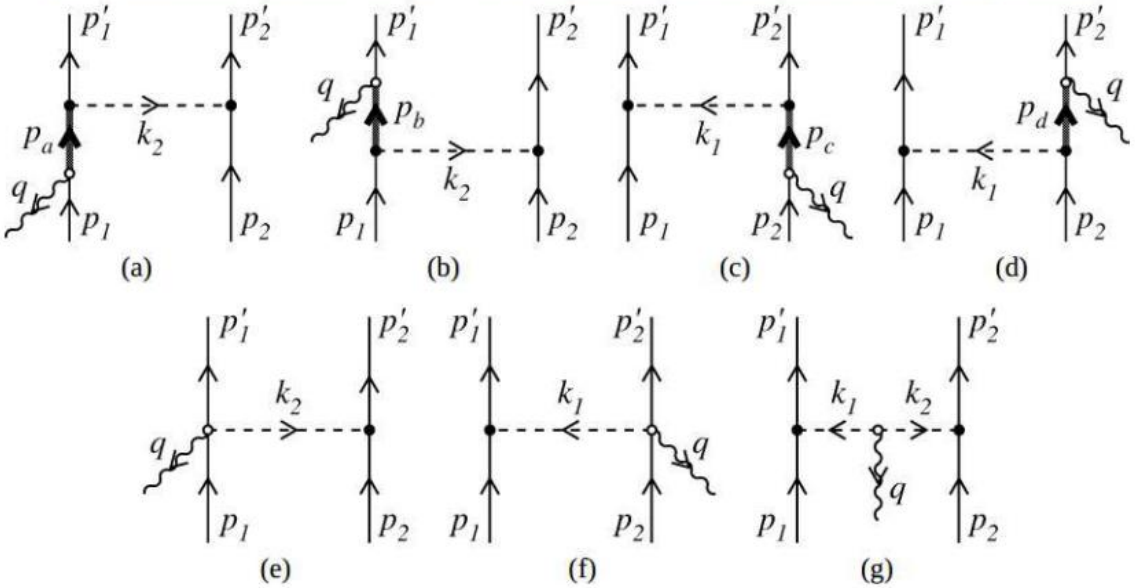




SuSAv2 2p2h

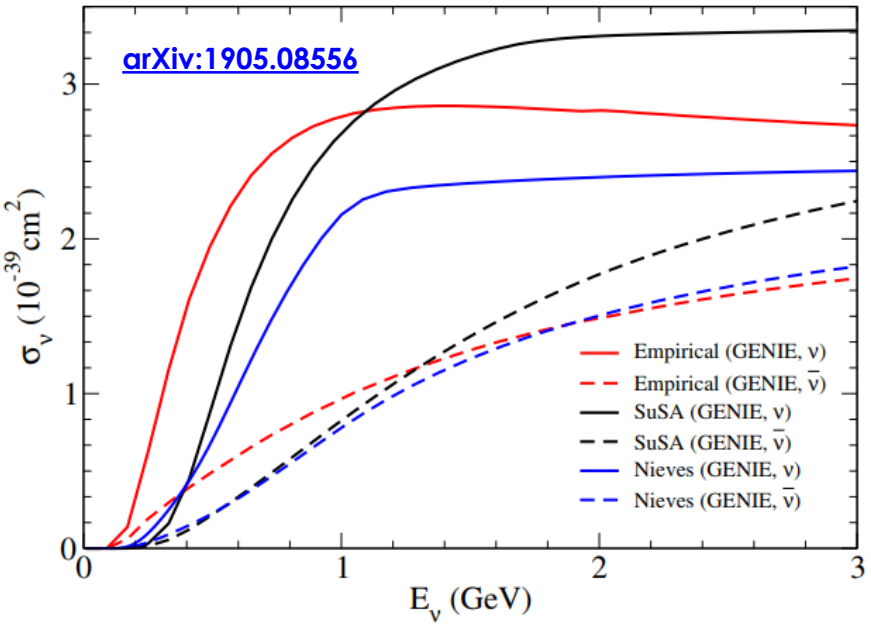
$$W^{\mu\nu}_{2p-2h} = \frac{V}{(2\pi)^9}\int d^3p'_1 d^3h_1 d^3h_2 \frac{M^4}{E_1 E_2 E'_1 E'_2}\Theta(p'_1, p'_2, h_1, h_2)$$
$$r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2)\delta(E'_1 + E'_2 - E_1 - E_2 - \omega)\,,$$

Over 100,000 terms are involved in the calculation, with seven-dimensional integrations



- Based on the calculation performed by De Pace et al., (2003) for (e, e') scattering and extended to the weak sector by Amaro, Ruiz Simo et al.
  [PRD 90, 033012 (2014); PRD 90, 053010 (2014); JPG 44, 065105 (2017); PLB 762, 124 (2016)]
- Performed within an RFG nuclear model (like Nieves), SuSAv2-MEC is fully relativistic – no approximations
- HUGE calculation, takes a long time to calculate a full cross section
  - Normally a parameterisation is used

Comparison to Valencia 2p2h



- Valencia model makes some non-relativistic approximations limiting validity above 1.2 GeV, SuSAv2-MEC does not.

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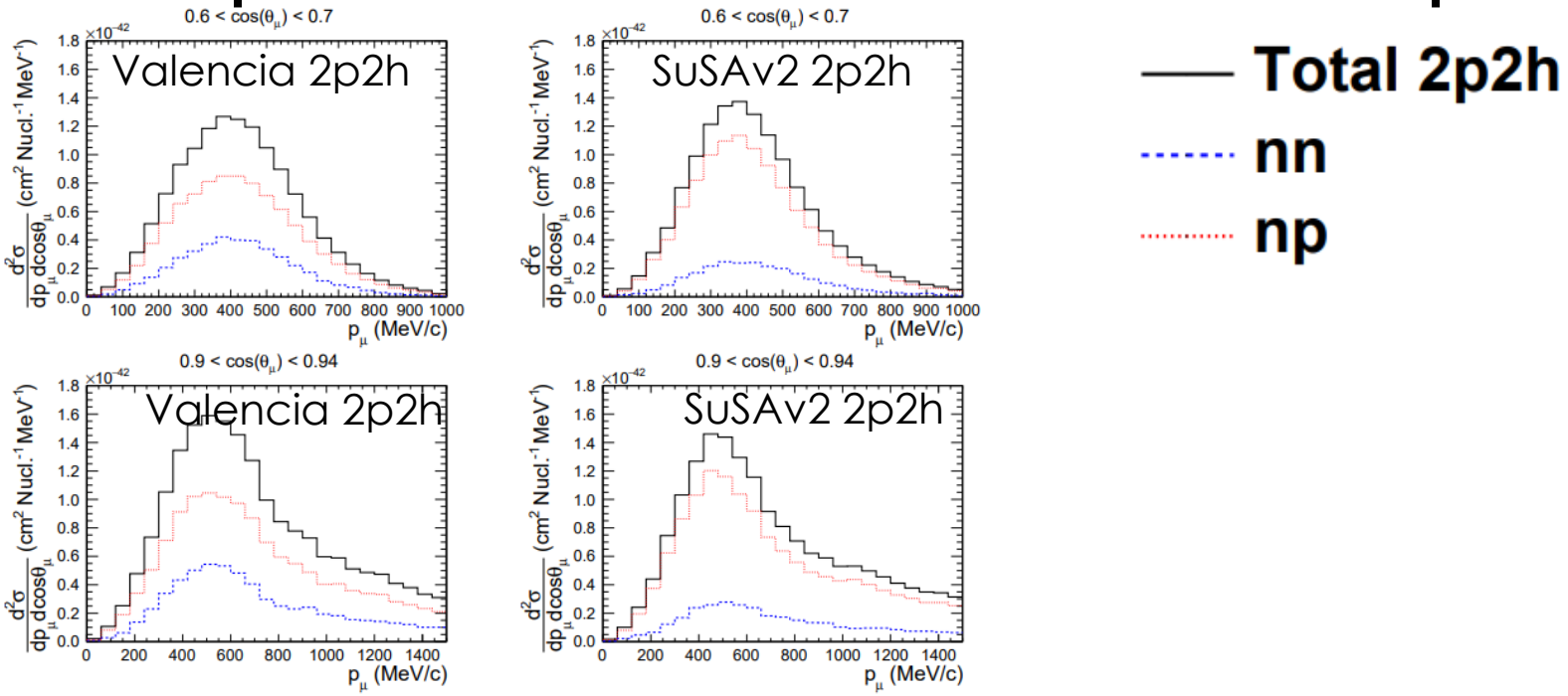

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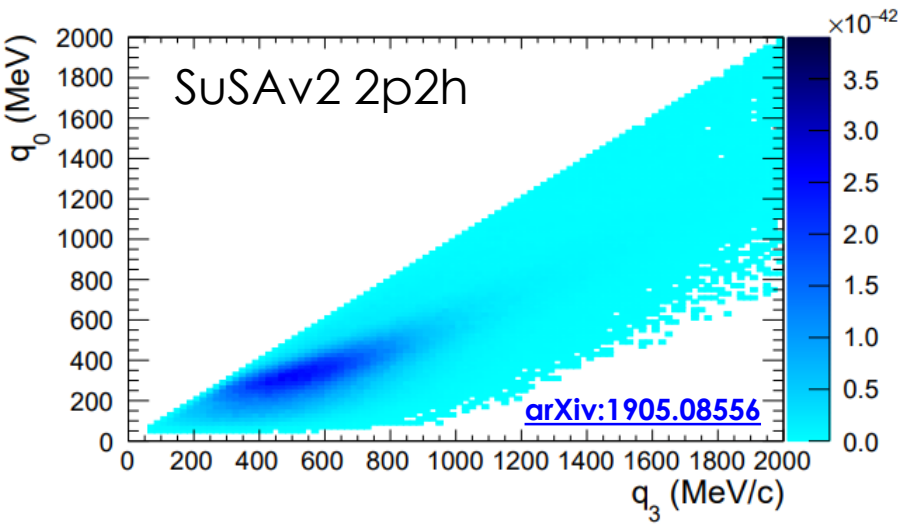
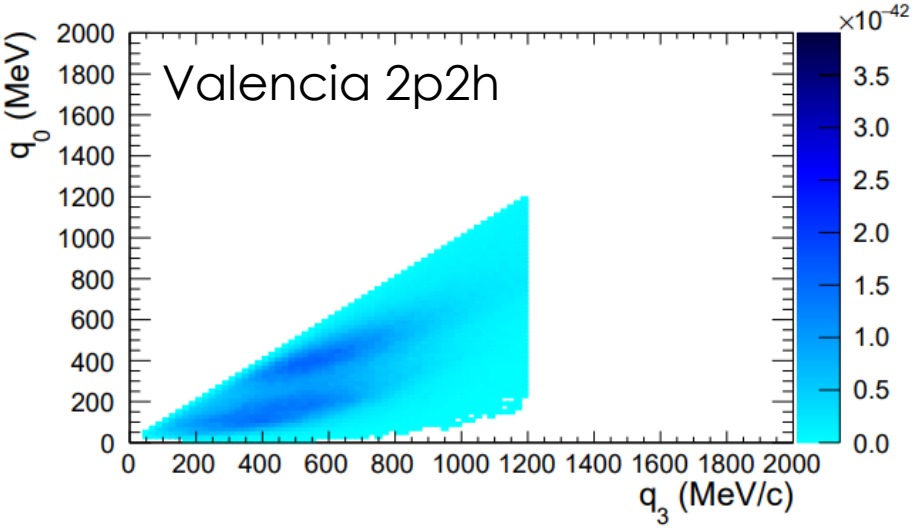
10.1016/j.physletb.2016.09.021

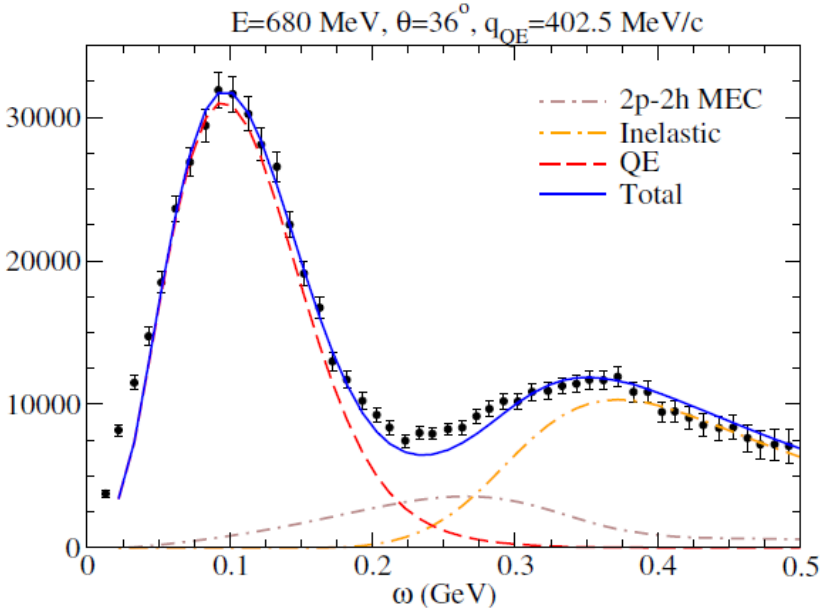
Comparison to Valencia 2p2h

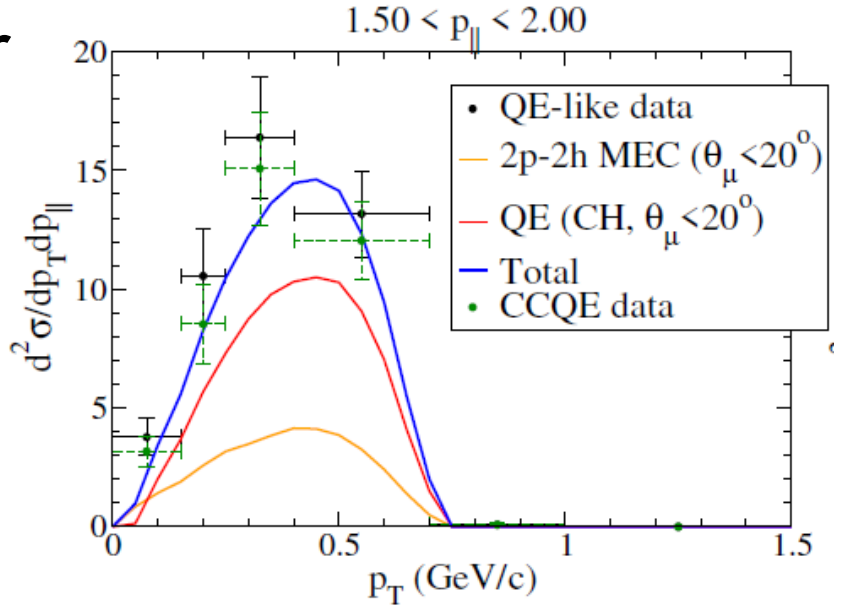


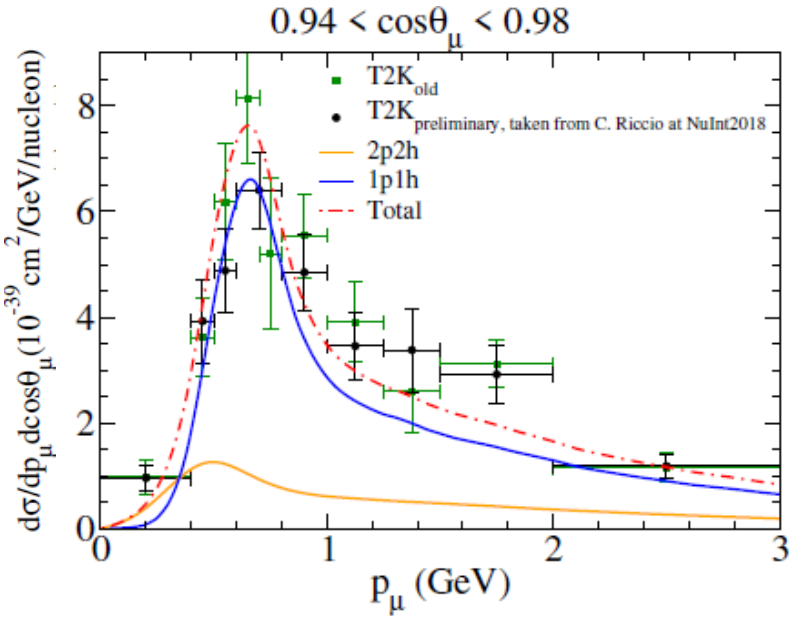


- Valencia model makes some non-relativistic approximations limiting validity above 1.2 GeV, SuSAv2-MEC does not.
- Valencia model rejects direct/exchange interference terms, SuSAv2-MEC does not – Valencia predicts relatively less pp final states
- Valencia model includes a different set of diagrams (some from imaginary part of the W)

Putting it together







- Based on sound microscopic model calculations
- **Well validated on electron scattering data**
- Is able to describe neutrino scattering data

Scaling variable definition

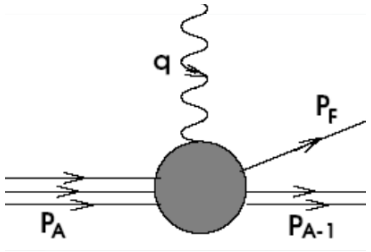
$$\psi' \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda' - \tau'}{\sqrt{(1+\lambda')\tau' + \kappa\sqrt{\tau'(\tau'+1)}}}$$

$$\lambda' = \omega'/(2M_N)\,, \quad \kappa = q/(2M_N)$$
$$\omega' = \omega - E_{shift}\,, \quad \tau' = \kappa^2 - \lambda'^2$$

Relativistic Mean Field (RMF)

Assumptions:

- **Mean** field: nucleons in an average potential – independent of each other. Correlations go into the potential, but are not explicit.
- Impulse approximation



Basic idea: bound and scattered nucleons are solutions of a Dirac-like equation w/ **energy independent** real scaler (attractive) and vector (repulsive) potentials.

$$(\tilde{E}\gamma_0 - \vec{p}\cdot\vec{\gamma} - \tilde{M})\psi = 0$$

$$\tilde{E} = E - V(r)$$
$$\tilde{M} = M - S(r)$$

As the final state nucleon feels a potential its wavefunction is distorted, this is how RMF handles FSI

Relativistic Mean Field (RMF)

RMF potentials are typically chosen to well describe the nuclear ground state (phenomenological potentials with inputs from data)

RMF also typically uses the same potentials for the initial and final state - but these are energy independent, so cannot work at all kinematics. See a problems at large q (FSI is too strong, should be very small).

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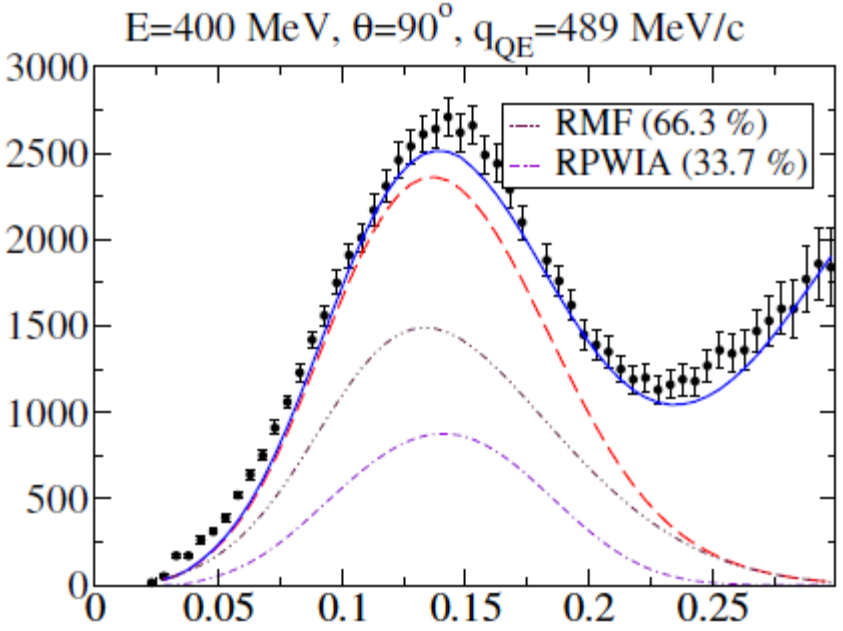
- SuSAv2 solution: introduce a kinematic dependent **blending function** between RMF and RPWIA (no FSI in final state)
- Future solutions: **ED-RMF** – incorporate same physics by adding energy dependence to the RMF potentials. WIP.



G. D. Megias PhD Thesis

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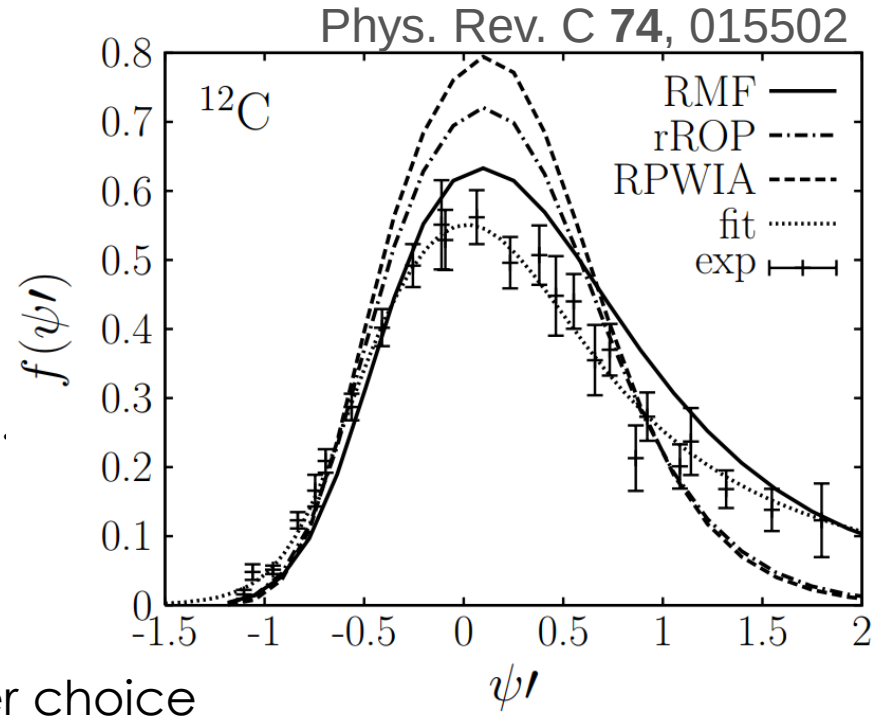
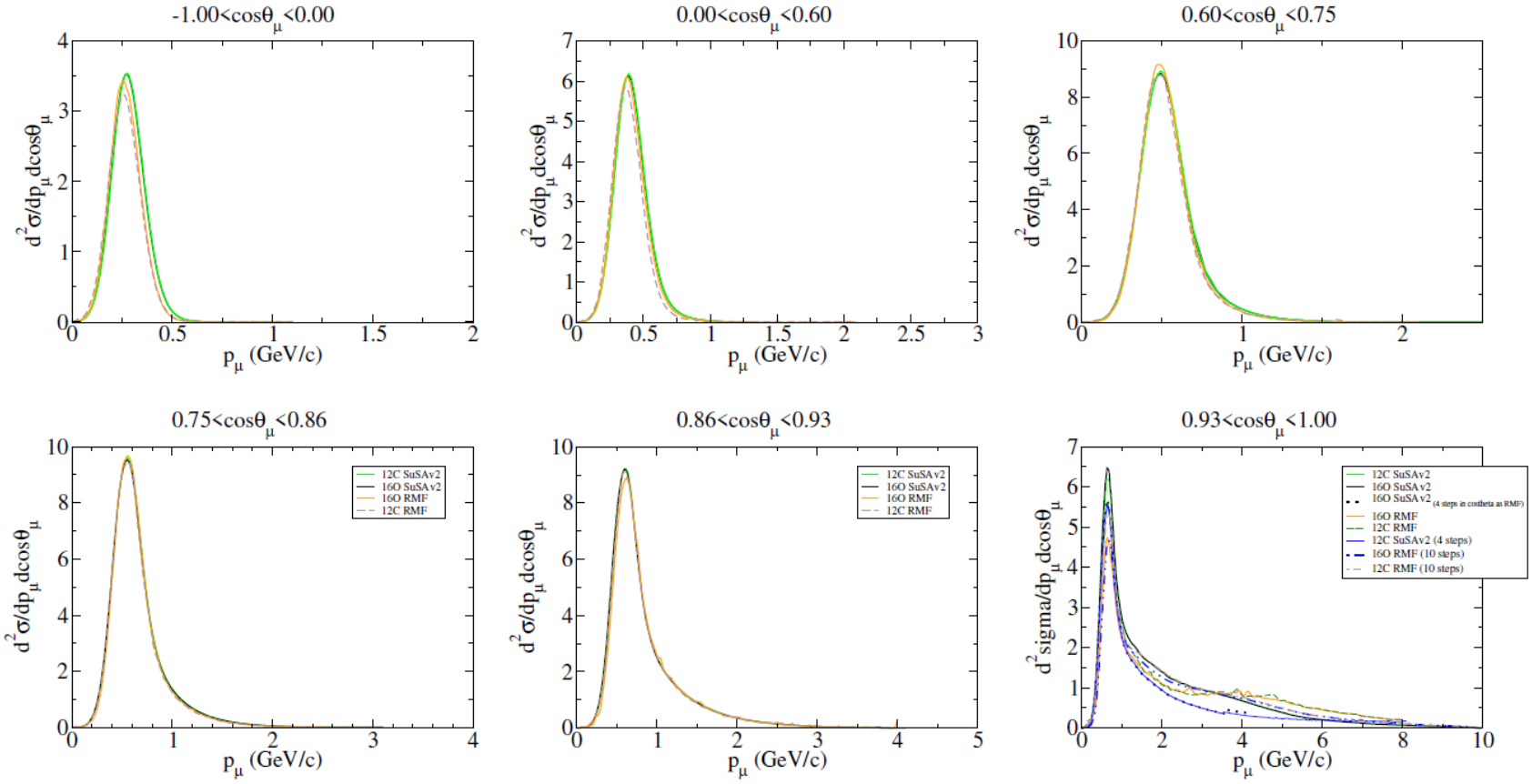
- SuSAv2 solution: introduce a kinematic dependent **blending function** between RMF and RPWIA (no FSI in final state)
- Future solutions: **ED-RMF** – incorporate same physics by adding energy dependence to the RMF potentials. WIP.
- Another option: use **optical potentials** (based on proton scattering data) to describe final state.
  - Often seems RMF potentials the better choice
  - Inconsistency: different final and initial state potentials …



RMF vs SuSAv2



Factorisation Approximation (FA)

https://indico.ectstar.eu/event/19/contributions/221/

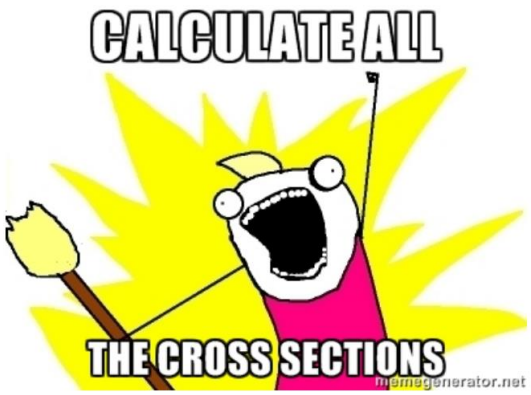
The inclusive models we use only have the hadron tensor elements needed to calculate lepton kinematics $\sigma \sim \eta_{\mu\nu} W^{\mu\nu}$

$$\eta^{s}_{\mu\nu} W^{\mu\nu}_{s} \sim \hat{V}_{CC} W^{CC}_{semi} + \hat{V}_{CL} W^{CL}_{semi} + \hat{V}_{LL} W^{LL}_{semi} + \hat{V}_{T} W^{T}_{semi} + \hat{V}_{TT} W^{TT}_{semi} + \hat{V}_{TC} W^{TC}_{semi} + \hat{V}_{TL} W^{TL}_{semi}$$

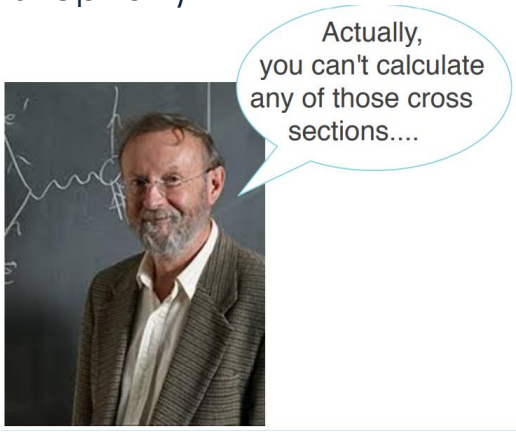
$$\eta^{a}_{\mu\nu} W^{\mu\nu}_{a} \sim \hat{V}_{T'} W^{T'}_{semi} + \hat{V}_{TC'} W^{TC'}_{semi} + \hat{V}_{TL'} W^{TL'}_{semi}$$

The terms needed to calculate hadrons **simply are not there**, everything we do to predict them is *ad-hoc* and should be treated with caution

Progression of the workshop (from G. Perdue's 2018 GENIE ECT* workshop talk)

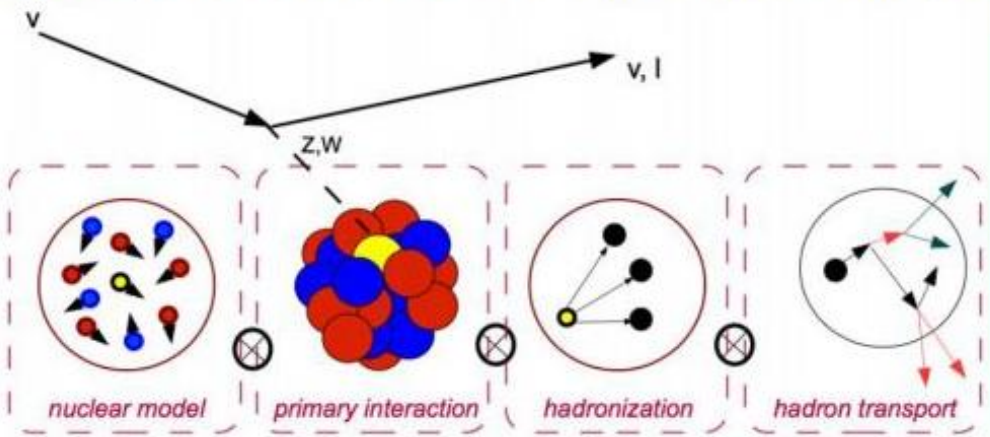


ALL the cross sections?



Factorisation Approximation (FA)

In the generators we factorise interactions



We use an inclusive ($\frac{d\sigma_{CCQE}}{dp_\mu dcos\theta_\mu}$) model for this bit. Only ad-hoc nuclear / FSI models then allow us to predict hadron kinematics

Theorists seem unconvinced …

- There is no guarantee of getting a result similar to a full semi-inclusive calculation:

$$\frac{d\sigma_{CCQE}}{dp_\mu d\theta_\mu dp_p d\theta_p d\theta_{\mu p}}$$

Like most 1p1h and 2p2h models in GENIE* SuSAv2-MEC models are **only valid for inclusive predictions**. When we try and use them to make semi-inclusive predictions we do **a lot of questionable physics**.

* Possibly excluding excluding Valencia 1p1h … ?

The ugly truth

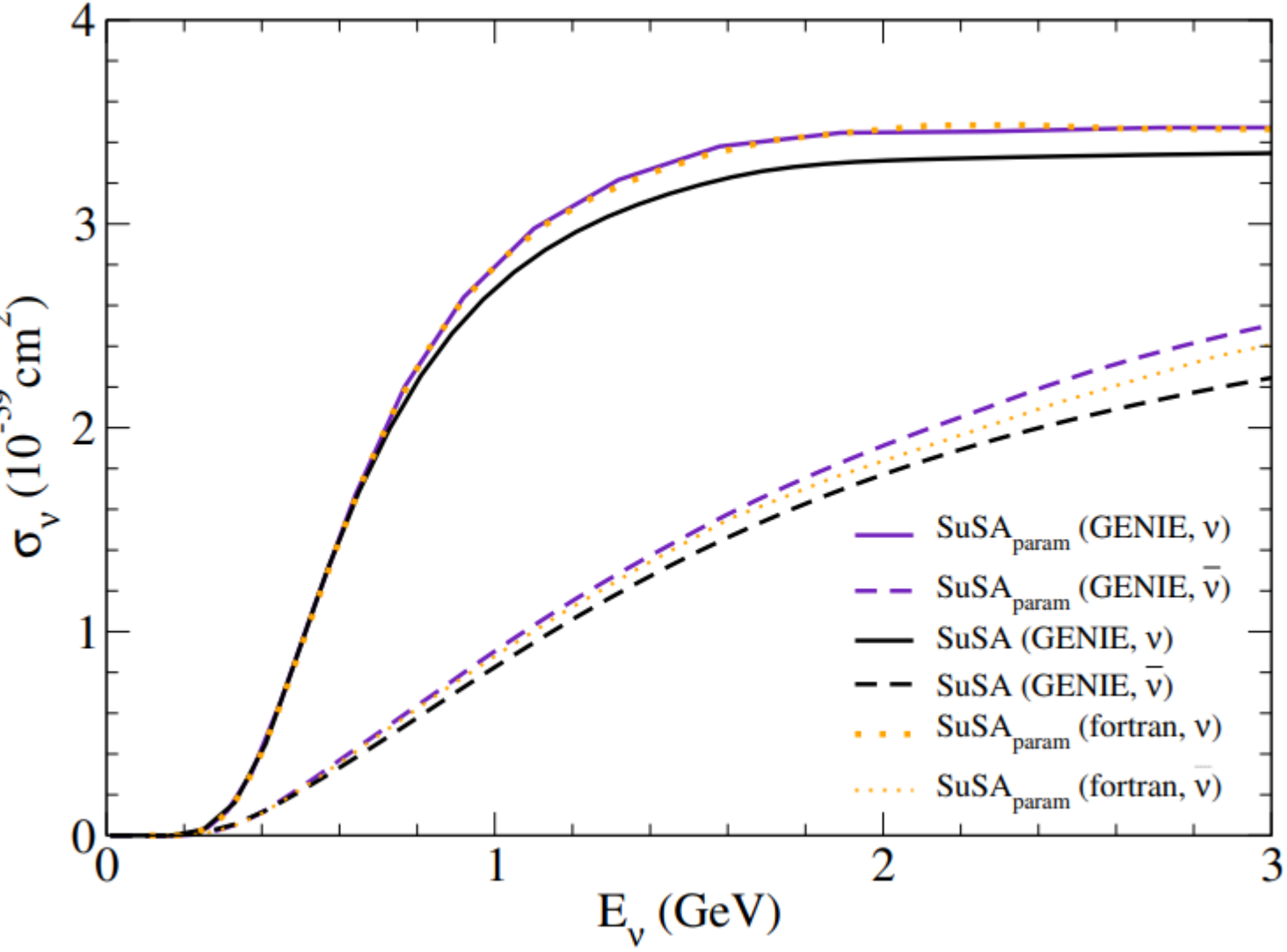
Assumptions to get hadron kinematics from inclusive models:

- *Inclusive interaction kinematics are independent of the initial-state nucleon kinematics and binding energy*
- *Semi-classical FSI-cascade, unrelated to the inclusive model's nuclear dynamics*
- *Simplistic treatment of energy(/momentum) transfer to the nuclear remnant (treatment depends on model)*

More general than just the impulse approximation

But without exclusive input model predictions, we can't do much better…

2p2h total xsec validations



Lots of scope for improvement!

Implementation of SuSAv2-MEC hadronic part (FA):

- Draw target nucleon from chosen nuclear model **irrespective of q_0, q_3**
- Get removal energy from RMF-like treatment, *re-throw from nuclear model if nucleon is Pauli blocked*
- Transfer all of ω, q to nucleon, **none to remnant**
- Subtract removal energy, **put proton on-shell with adjustment of p** (only needed for 1p1h) **then conserve momentum by adjusting remnant kinematics**
- *Do FSI cascade and rest of interaction using standard GENIE methods*

Lots of scope for improvement!

Avoid by sampling full exclusive xsec?

Mitigate by making nuclear model q_0, q_3 dependant?

Different model for each shell?

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Removal energy should also depend on chosen initial nucleon momentum which should depend on inclusive kinematics …

The remnant should take *some* momentum … how much?

This seems really bad … but if we sample a 2D inclusive xsec and choose the nucleon momentum in a factorised way can we ever do much better?

How much can we improve FSI? Better motivated cascades (INCL, maybe)? GiBUU-like hadron transport?

Lots of scope for improvement!

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Implen… SAv2-MEC hadronic part (FA):

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Need to use more exclusive inputs from theory!

Future work – SuSAv2

- The only major improvement to the current implementation foreseen is to add new hadron tensors for relevant targets (Argon in particular)
    - This is easy for SuSAv2, but for the 2p2h requires theorists to re-run a very computationally intensive code (weeks/months)
- Addressing this in the current implementation is like polishing the brass on the Titanic…